Dividing Positive and Negative Fractions

Main IDEA

Divide positive and negative fractions.

Targeted TEKS 8.2
The student selects and uses appropriate operations to solve problems and justify solutions. **(B) Use appropriate operations to solve problems involving rational numbers in problem situations.**

NEW Vocabulary

multiplicative inverses
reciprocals

Concepts in Motion
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▶ GET READY for the Lesson

ANIMALS The world's longest snake is the reticulated python. It is approximately one-fourth the length of the blue whale.

1. Find the value of $110 \div 4$.

2. Find the value of $110 \times \frac{1}{4}$.

3. Compare the values of $110 \div 4$ and $110 \times \frac{1}{4}$.

4. What can you conclude about the relationship between dividing by 4 and multiplying by $\frac{1}{4}$?

World's Largest Animals		
Largest Animal	Blue Whale	110 feet long
Largest Reptile	Saltwater Crocodile	16 feet long
Largest Bird	Ostrich	9 feet tall
Largest Insect	Stick Insect	15 inches long

Source: *The World Almanac for Kids*

Two numbers whose product is 1 are **multiplicative inverses**, or **reciprocals**, of each other. For example, 4 and $\frac{1}{4}$ are multiplicative inverses because $4 \cdot \frac{1}{4} = 1$.

KEY CONCEPT
Inverse Property of Multiplication

Words   The product of a number and its multiplicative inverse is 1.

Examples

Numbers	Algebra
$\frac{3}{4} \cdot \frac{4}{3} = 1$	$\frac{a}{b} \cdot \frac{b}{a} = 1$, where a and $b \neq 0$

EXAMPLE   Find a Multiplicative Inverse

1 Write the multiplicative inverse of $-5\frac{2}{3}$.

$-5\frac{2}{3} = -\frac{17}{3}$ Write $-5\frac{2}{3}$ as an improper fraction.

Since $-\frac{17}{3}\left(-\frac{3}{17}\right) = 1$, the multiplicative inverse of $-5\frac{2}{3}$ is $-\frac{3}{17}$.

✓ CHECK Your Progress

Write the multiplicative inverse of each number.

a. $-2\frac{1}{3}$ b. $-\frac{5}{8}$ c. 7

Dividing by 4 is the same as multiplying by $\frac{1}{4}$, its multiplicative inverse. This is true for any rational number.

multiplicative inverses

$$110 \div 4 = 27\frac{1}{2} \qquad 110 \cdot \frac{1}{4} = 27\frac{1}{2}$$

KEY CONCEPT   Divide Fractions

Words   To divide by a fraction, multiply by its multiplicative inverse.

Examples   **Numbers**   **Algebra**

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \cdot \frac{4}{3} \qquad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}, \text{ where } b, c, \text{ and } d \neq 0$$

EXAMPLES   Divide Fractions

Divide. Write in simplest form.

② $\frac{7}{8} \div \frac{3}{4}$

$\frac{7}{8} \div \frac{3}{4} = \frac{7}{8} \cdot \frac{4}{3}$ Multiply by the multiplicative inverse of $\frac{3}{4}$, which is $\frac{4}{3}$.

$= \frac{7}{\overset{}{\underset{2}{8}}} \cdot \frac{\overset{1}{4}}{3}$ Divide 8 and 4 by their GCF, 4.

$= \frac{7}{6}$ or $1\frac{1}{6}$ Multiply.

③ $-\frac{4}{5} \div \frac{6}{7}$

$-\frac{4}{5} \div \frac{6}{7} = -\frac{4}{5} \cdot \frac{7}{6}$ Multiply by the multiplicative inverse of $\frac{6}{7}$, which is $\frac{7}{6}$.

$= -\frac{\overset{2}{4}}{5} \cdot \frac{7}{\underset{3}{6}}$ Divide −4 and 6 by their GCF, 2.

$= -\frac{14}{15}$ Multiply

✓ **CHECK Your Progress**   **Divide. Write in simplest form.**

d. $\frac{3}{4} \div \frac{1}{2}$ e. $-\frac{1}{4} \div \frac{7}{8}$ f. $-\frac{2}{3} \div \left(-\frac{3}{5}\right)$

EXAMPLE   Divide by a Whole Number

④ Find $\frac{2}{5} \div 5$. Write in simplest form.

$\frac{2}{5} \div 5 = \frac{2}{5} \div \frac{5}{1}$ Write 5 as $\frac{5}{1}$.

$= \frac{2}{5} \cdot \frac{1}{5}$ or $\frac{2}{25}$ Multiply by the multiplicative inverse of 5, which is $\frac{1}{5}$.

✓ **CHECK Your Progress**   **Divide. Write in simplest form.**

g. $\frac{4}{9} \div 3$ h. $-\frac{3}{5} \div 6$ i. $-\frac{9}{16} \div (-12)$

5 Find $4\frac{2}{3} \div \left(-3\frac{1}{2}\right)$. Write in simplest form.

Estimate $5 \div (-4) = \frac{5}{-4}$ or $-1\frac{1}{4}$

$$4\frac{2}{3} \div \left(-3\frac{1}{2}\right) = \frac{14}{3} \div \left(-\frac{7}{2}\right) \qquad 4\frac{2}{3} = 1\frac{4}{3}, \; -3\frac{1}{2} = -\frac{7}{2}$$

$$= \frac{14}{3} \cdot \left(-\frac{2}{7}\right) \qquad \text{The multiplicative inverse of } -\frac{7}{2} \text{ is } -\frac{2}{7}.$$

$$= \frac{\overset{2}{14}}{3} \cdot \left(-\frac{2}{\underset{1}{7}}\right) \qquad \text{Divide 14 and 7 by their GCF, 7.}$$

$$= -\frac{4}{3} \text{ or } -1\frac{1}{3} \qquad \text{Multiply.}$$

Check for Reasonableness Compare to the estimate. The answer seems reasonable because $-1\frac{1}{3}$ is about $-1\frac{1}{4}$. ✔

CHECK Your Progress

Divide. Write in simplest form.

j. $2\frac{3}{4} \div \left(-2\frac{1}{5}\right)$ 　　　 k. $1\frac{1}{2} \div 2\frac{1}{3}$ 　　　 l. $-3\frac{1}{2} \div \left(-1\frac{1}{4}\right)$

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Real-World Link . . . .
The first Flag Day was celebrated in 1877. It was the 100th anniversary of the day the Continental Congress adopted the Stars and Stripes as the official flag.
Source: *World Book*

Real-World EXAMPLE

6 **HOLIDAYS** Isabel and her friends are making ribbons to give to other campers at their day camp on Flag Day. They have a roll with 20 feet of ribbon. How many Flag Day ribbons as shown at the right can they make?

Since 4 inches equals $\frac{4}{12}$ or $\frac{1}{3}$ foot, divide 20 by $\frac{1}{3}$.

$$20 \div \frac{1}{3} = \frac{20}{1} \div \frac{1}{3} \qquad \text{Write 20 as } \frac{20}{1}.$$

$$= \frac{20}{1} \cdot \frac{3}{1} \qquad \text{Multiply by the multiplicative inverse of } \frac{1}{3}, \text{ which is 3.}$$

$$= \frac{60}{1} \text{ or } 60 \qquad \text{Simplify.}$$

Isabel and her friends can make 60 Flag Day ribbons.

STUDY TIP

Mental Math Isabel can make 3 ribbons for each foot. Since 3×20 is 60, Isabel can make 60 ribbons.

CHECK Your Progress

m. **SHOPPING** Tito has $20 to buy apples at the farmer's market. If the market charges $2 for 3 apples, how many apples can Tito buy?

n. **ENERGY** Electricity costs $6\frac{1}{2}$¢ per kilowatt-hour. Of that cost, $3\frac{1}{4}$¢ goes toward the cost of the fuel. What fraction of the cost goes towards fuel?

CHECK Your Understanding

Example 1
(p. 102)

Write the multiplicative inverse of each number.

1. $\dfrac{5}{7}$

2. -12

3. $-2\dfrac{3}{4}$

Examples 2, 3
(p. 103)

Divide. Write in simplest form.

4. $\dfrac{2}{3} \div \dfrac{3}{4}$

5. $\dfrac{5}{8} \div \dfrac{1}{2}$

6. $\dfrac{3}{8} \div \left(-\dfrac{9}{10}\right)$

7. $-\dfrac{7}{16} \div \left(-\dfrac{7}{8}\right)$

Examples 4, 5
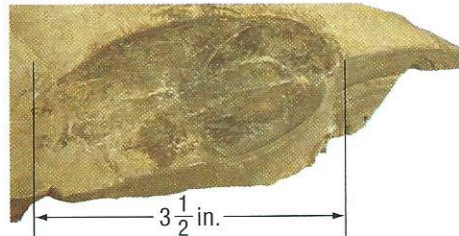(pp. 103–104)

8. $\dfrac{4}{5} \div 8$

9. $\dfrac{9}{10} \div 3$

10. $-5\dfrac{5}{6} \div \left(-4\dfrac{2}{3}\right)$
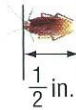
11. $-3\dfrac{7}{12} \div 6\dfrac{5}{6}$

Example 6
(p. 104)

12. **BIOLOGY** The 300 million-year-old fossil of a cockroach was recently found in eastern Ohio. The ancient cockroach is shown next to the common German cockroach found today.

300-Million-Year-Old Cockroach

Common German Cockroach

$\frac{1}{2}$ in.

$3\frac{1}{2}$ in.

How many times longer is the ancient cockroach than the German cockroach?

Exercises

Write the multiplicative inverse of each number.

13. $-\dfrac{7}{9}$

14. $-\dfrac{5}{8}$

15. 15

16. 18

17. $3\dfrac{2}{5}$

18. $4\dfrac{1}{8}$

Divide. Write in simplest form.

19. $\dfrac{2}{5} \div \dfrac{3}{4}$

20. $\dfrac{3}{8} \div \dfrac{2}{3}$

21. $\dfrac{2}{3} \div \dfrac{5}{6}$

22. $\dfrac{2}{5} \div \dfrac{1}{10}$

23. $-\dfrac{4}{5} \div \dfrac{3}{4}$

24. $\dfrac{3}{10} \div \left(-\dfrac{2}{3}\right)$

25. $-\dfrac{5}{9} \div \left(-\dfrac{2}{3}\right)$

26. $-\dfrac{7}{12} \div \left(-\dfrac{5}{6}\right)$

27. $\dfrac{2}{5} \div 4$

28. $\dfrac{9}{16} \div 3$
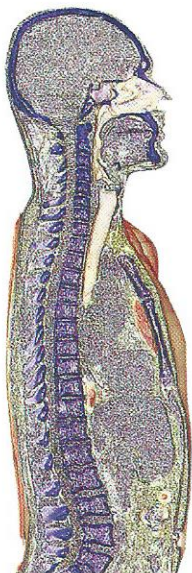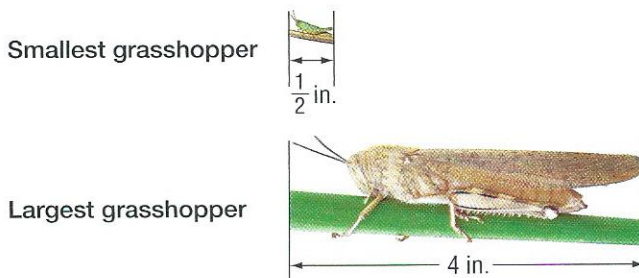
29. $\dfrac{4}{5} \div 6$

30. $\dfrac{6}{7} \div 4$

31. $3\dfrac{3}{4} \div 2\dfrac{1}{2}$

32. $7\dfrac{1}{2} \div 2\dfrac{1}{10}$

33. $-12\dfrac{1}{4} \div 4\dfrac{2}{3}$

34. $10\dfrac{1}{5} \div \left(-\dfrac{3}{15}\right)$

35. **BIOLOGY** Use the information below. How many of the smallest grasshoppers need to be laid end-to-end to have the same length as one of the largest grasshoppers?

Smallest grasshopper $\frac{1}{2}$ in.

Largest grasshopper ——— 4 in. ———

36. **LIBRARIES** Pilar is storing a set of art books on a shelf that has $11\frac{1}{4}$ inches of shelf space. If each book is $\frac{3}{4}$ inch wide, how many books can be stored on the shelf?

HUMAN BODY For Exercises 37 and 38, use the information below and at the right.

The table shows the composition of a healthy adult male's body. Examples of body cell mass are muscle, body organs, and blood. Examples of supporting tissue are blood plasma and bones.

Composition of Human Body	
Component	**Fraction of Body Weight**
Body Cell Mass	$\frac{11}{20}$
Supporting Tissue	$\frac{3}{10}$
Body Fat	$\frac{3}{20}$

Source: about.com

37. How many times more of a healthy adult male's body weight is made up of body cell mass than body fat?

38. How many times more of a healthy adult male's body weight is made up of body cell mass than supporting tissue?

39. **GEOMETRY** The circumference C, or distance around a circle, can be approximated using the formula $C = \frac{44}{7}r$, where r is the radius of the circle. What is the radius of the circle at the right? Round to the nearest tenth.
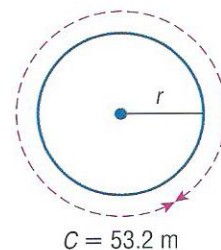
r

$C = 53.2$ m

40. **BAKING** Emily is baking chocolate cupcakes. Each batch of 20 cupcakes requires $\frac{2}{3}$ cups of cocoa. If Emily has $3\frac{1}{4}$ cups of cocoa, how many full batches of cupcakes will she be able to make and how much cocoa will she have left over?

H.O.T. Problems

41. **OPEN ENDED** Select a fraction between 0 and 1. Identify both its additive and multiplicative inverses. Explain your reasoning.

42. **CHALLENGE** Give a counterexample to the statement *The quotient of two fractions between 0 and 1 is never a whole number.*

43. NUMBER SENSE Which is greater: $30 \cdot \frac{3}{4}$ or $30 \div \frac{3}{4}$? Explain.
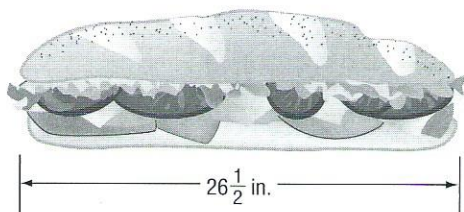
CHALLENGE Use mental math to find each value.

44. $\frac{43}{594} \cdot \frac{641}{76} \div \frac{641}{594}$

45. $\frac{783}{241} \cdot \frac{241}{783} \div \frac{72}{53}$

46. **WRITING IN MATH** Write a real-world problem that can be solved by dividing fractions or mixed numbers. Solve the problem.

TEST PRACTICE

47. A submarine sandwich that is $26\frac{1}{2}$ inches long is cut into $4\frac{5}{12}$-inch mini-subs. How many mini-subs are there?

$\vdash \quad 26\frac{1}{2} \text{ in.} \quad \dashv$

A 4   **C** 6

B 5   **D** 7

48. Mr. Jones is doing a science experiment with his class of 20 students. Each student needs $\frac{3}{4}$ cup of vinegar. If he currently has 15 cups of vinegar, which equation could Mr. Jones use to determine if he has enough vinegar for his entire class?

F $x = 15 \div 20$

G $x = 15 \div \frac{3}{4}$

H $x = 20 - (15)$

J $x = 15(20)$

Spiral Review

Multiply. Write in simplest form. (Lesson 2-3)

49. $\frac{1}{2} \cdot \frac{3}{4}$

50. $\frac{7}{12} \cdot \frac{4}{7}$

51. $1\frac{2}{3} \cdot 4\frac{1}{5}$

52. $\frac{2}{3} \cdot 3\frac{1}{4}$

53. SCHOOL In a survey of students at Centerburg Middle School, $\frac{13}{20}$ of the boys and $\frac{17}{25}$ of the girls said they ride the bus to school. Of those surveyed, do a greater fraction of boys or girls ride the bus? (Lesson 2-2)

54. ALGEBRA Write an equation using two variables that could be used to determine the population of Asia if it is about three million less than five times the population of Africa. (Lesson 1-7)

Write an integer to describe each situation. (Lesson 1-3)

55. 10 candy bars short of his goal

56. 7 bonus points

GET READY for the Next Lesson

PREREQUISITE SKILL Add or subtract. (Lessons 1-4 and 1-5)

57. $-7 + 15$

58. $-9 + (-4)$

59. $-3 - 15$

60. $12 - (-17)$