Multiplying Positive and Negative Fractions

NEW Vocabulary
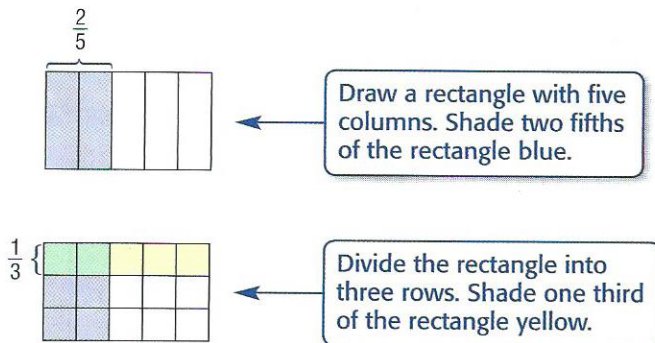
dimensional analysis

▶ MINI Lab

COncepts in MOtion
Animation tx.msmath3.com

To multiply $\frac{1}{3}$ and $\frac{2}{5}$, you can use an area model to find $\frac{1}{3}$ of $\frac{2}{5}$.

$\frac{2}{5}$

> Draw a rectangle with five columns. Shade two fifths of the rectangle blue.

$\frac{1}{3}$ {

> Divide the rectangle into three rows. Shade one third of the rectangle yellow.

The green shaded area represents $\frac{1}{3}$ of $\frac{2}{5}$.

1. What is the product of $\frac{1}{3}$ and $\frac{2}{5}$?

2. Use an area model to find each product.

   a. $\frac{3}{4} \cdot \frac{1}{2}$ b. $\frac{2}{5} \cdot \frac{2}{3}$

   c. $\frac{1}{4} \cdot \frac{3}{5}$ d. $\frac{2}{3} \cdot \frac{4}{5}$

3. What is the relationship between the numerators of the factors and the numerator of the product?

4. What is the relationship between the denominators of the factors and the denominator of the product?

The Mini Lab suggests the rule for multiplying fractions.

KEY CONCEPT   Multiply Fractions

Words   To multiply fractions, multiply the numerators and multiply the denominators.

Examples   **Numbers**   **Algebra**

$\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$ $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ where b and $d \neq 0$

You can use the rules for multiplying integers to determine the sign of the product of any two signed numbers.

EXAMPLE **Multiply Positive Fractions**

① Find $\dfrac{4}{9} \cdot \dfrac{3}{5}$. Write in simplest form.

$$\dfrac{4}{9} \cdot \dfrac{3}{5} = \dfrac{4}{\cancel{9}} \cdot \dfrac{\cancel{3}^{1}}{5} \qquad \text{Divide 9 and 3 by their GCF, 3.}$$

$$= \dfrac{4 \cdot 1}{3 \cdot 5} \qquad \begin{array}{l} \leftarrow \text{Multiply the numerators.} \\ \leftarrow \text{Multiply the denominators.} \end{array}$$

$$= \dfrac{4}{15} \qquad \text{Simplify.}$$

CHECK Your Progress Multiply. Write in simplest form.

a. $\dfrac{1}{4} \cdot \dfrac{2}{3}$ b. $\dfrac{5}{12} \cdot \dfrac{3}{20}$ c. $\dfrac{7}{10} \cdot \dfrac{7}{16}$

EXAMPLE **Multiply Negative Fractions**

② Find $-\dfrac{5}{6} \cdot \dfrac{3}{8}$. Write in simplest form.

$$-\dfrac{5}{6} \cdot \dfrac{3}{8} = \dfrac{-5}{\cancel{6}_{2}} \cdot \dfrac{\cancel{3}^{1}}{8} \qquad \text{Divide 6 and 3 by their GCF, 3.}$$

$$= \dfrac{-5 \cdot 1}{2 \cdot 8} \qquad \begin{array}{l} \leftarrow \text{Multiply the numerators.} \\ \leftarrow \text{Multiply the denominators.} \end{array}$$

$$= -\dfrac{5}{16} \qquad \begin{array}{l} \text{The fractions have different signs,} \\ \text{so the product is negative.} \end{array}$$

CHECK Your Progress Multiply. Write in simplest form.

d. $\dfrac{8}{9} \cdot -\dfrac{3}{4}$ e. $-\dfrac{3}{5} \cdot \dfrac{7}{9}$ f. $\left(-\dfrac{1}{2}\right)\left(-\dfrac{6}{7}\right)$

To multiply mixed numbers, first rename them as improper fractions.

EXAMPLE **Multiply Mixed Numbers**

③ Find $4\dfrac{1}{2} \cdot 2\dfrac{2}{3}$. Write in simplest form. **Estimate** $4 \times 3 = 12$

$$4\dfrac{1}{2} \cdot 2\dfrac{2}{3} = \dfrac{9}{2} \cdot \dfrac{8}{3} \qquad 4\dfrac{1}{2} = \dfrac{9}{2},\ 2\dfrac{2}{3} = \dfrac{8}{3}$$

$$= \dfrac{\cancel{9}^{3}}{\cancel{2}_{1}} \cdot \dfrac{\cancel{8}^{4}}{\cancel{3}_{1}} \qquad \text{Divide out common factors.}$$

$$= \dfrac{3 \cdot 4}{1 \cdot 1} \qquad \begin{array}{l} \leftarrow \text{Multiply the numerators.} \\ \leftarrow \text{Multiply the denominators.} \end{array}$$

$$= \dfrac{12}{1} \text{ or } 12 \qquad \text{Simplify. Compare to the estimate.}$$

CHECK Your Progress Multiply. Write in simplest form.

g. $1\dfrac{1}{2} \cdot 1\dfrac{2}{3}$ h. $\dfrac{5}{7} \cdot 1\dfrac{3}{5}$ i. $\left(-2\dfrac{1}{6}\right)\left(-1\dfrac{1}{5}\right)$

Real-World EXAMPLE

4 **ROLLER COASTERS** A roller coaster at an amusement park is 160 feet high. If a new roller coaster is built that is $2\frac{3}{5}$ times the height of the existing coaster, what is the height of the new roller coaster?

The new coaster is $2\frac{3}{5}$ times higher than the current coaster.

$$2\frac{3}{5} \cdot 160 = \frac{13}{5} \cdot \frac{160}{1} \qquad 2\frac{3}{5} = \frac{13}{5},\ 160 = \frac{160}{1}$$

$$= \frac{2,080}{5} \text{ or } 416 \qquad \text{The new roller coaster will be 416 feet high.}$$

✓ CHECK Your Progress

j. CARPENTRY A piece of lumber is $4\frac{1}{4}$ feet long. If you need a piece of lumber that is $\frac{2}{3}$ this size, how long a piece do you need?

Real-World Link . . . . .
A 757 aircraft has an average cruising speed of 540 miles per hour, a capacity of 242 passengers, and a wingspan of $165\frac{1}{3}$ feet.

Source: Continental Traveler

Dimensional analysis is the process of including units of measurement when you compute. You can use dimensional analysis to check whether your answers are reasonable.

EXAMPLE Use Dimensional Analysis

5 **AIRCRAFT** Refer to the information at the left. Suppose a 757 aircraft is traveling at its cruising speed. How far will it travel in $1\frac{1}{3}$ hours?

Words	Distance equals the rate multiplied by the time.
Variable	Let d represent the distance.
Equation	$d = 540$ miles per hour $\cdot\ 1\frac{1}{3}$ hours

STUDY TIP

Mental Math
$\frac{1}{3}$ of 540 is 180.
Using the Distributive Property, $1\frac{1}{3}$ of 540 should equal
$540 + 180$, or 720.

$d = \dfrac{540 \text{ miles}}{1 \text{ hour}} \cdot 1\frac{1}{3}$ hours   Write the equation.

$d = \dfrac{540 \text{ miles}}{1 \text{ hour}} \cdot \dfrac{4}{3} \cdot \dfrac{\text{hours}}{1}$ $1\frac{1}{3} = \frac{4}{3}$

$d = \dfrac{\overset{180}{\cancel{540}} \text{ miles}}{1\ \cancel{\text{hour}}} \cdot \dfrac{4}{\underset{1}{\cancel{3}}} \cdot \dfrac{\cancel{\text{hours}}}{1}$ Divide by common factors and units.

$d = 720$ miles

At its cruising speed, a 757 will travel 720 miles in $1\frac{1}{3}$ hours.

Check The problem asks for the distance. When you divide the common units, the answer is expressed in miles.

✓ CHECK Your Progress

k. AIRCRAFT Refer to the information about the 757 aircraft. What is its wingspan in yards?

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Examples 1–3
(p. 97)

Multiply. Write in simplest form.

1. $\dfrac{3}{5} \cdot \dfrac{5}{7}$

2. $\dfrac{4}{5} \cdot \dfrac{3}{8}$

3. $\dfrac{6}{7} \cdot \dfrac{7}{6}$

4. $-\dfrac{1}{8} \cdot \dfrac{4}{9}$

5. $-\dfrac{2}{9} \cdot \left(\dfrac{3}{8}\right)$

6. $\left(-\dfrac{12}{13}\right)\left(-\dfrac{2}{3}\right)$
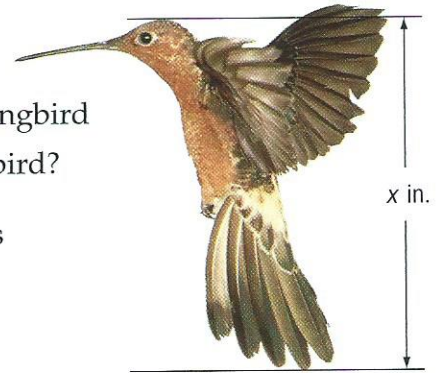
7. $1\dfrac{1}{3} \cdot 5\dfrac{1}{2}$

8. $2\dfrac{1}{2} \cdot 1\dfrac{2}{5}$

9. $-6\dfrac{3}{4} \cdot 1\dfrac{7}{9}$

Example 4
(p. 98)

10. **BIOLOGY** The giant hummingbird of South America is the largest hummingbird in the world. It is $4\dfrac{1}{8}$ times larger than the bee hummingbird. If the length of a bee hummingbird is 2 inches, how long is the giant hummingbird?

Giant Hummingbird

x in.

Example 5
(p. 98)

11. **FRUIT** Terrence bought $2\dfrac{5}{8}$ pounds of grapes that cost $2 per pound. What was the total cost of the grapes?

Exercises

For Exercises	See Examples
12–15	1
16–19	2
20–23	3
24, 25	4
26–29	5

HOMEWORK HELP

Multiply. Write in simplest form.

12. $\dfrac{1}{12} \cdot \dfrac{4}{7}$

13. $\dfrac{3}{16} \cdot \dfrac{1}{9}$

14. $\dfrac{5}{8} \cdot \dfrac{4}{5}$

15. $\dfrac{9}{10} \cdot \dfrac{2}{3}$

16. $-\dfrac{9}{10} \cdot \dfrac{2}{3}$

17. $\left(-\dfrac{12}{25}\right)\dfrac{15}{32}$

18. $\left(-\dfrac{3}{5}\right)\left(-\dfrac{1}{3}\right)$

19. $\left(-\dfrac{4}{7}\right)\left(-\dfrac{1}{20}\right)$

20. $3\dfrac{1}{3} \cdot \dfrac{1}{4}$

21. $4\dfrac{1}{4} \cdot 3\dfrac{1}{3}$

22. $-3\dfrac{3}{8} \cdot \left(-\dfrac{2}{3}\right)$

23. $-\dfrac{5}{6} \cdot \left(-1\dfrac{4}{5}\right)$

24. **FOOD** The nutrition label is from a can of green beans. How many cups of green beans does the can contain?

25. **PHOTOGRAPHY** Minh-Thu has a square photograph of the volleyball team that measures $3\dfrac{1}{2}$ inches on each side. She reduces each dimension to $\dfrac{2}{3}$ its size. What is the length of a side of the new photograph?

Nutrition Facts
Serving Size ¹/₂ cup (121g)
Servings Per Container approx. 3¹/₂

Amount Per Serving
Calories 20 Calories from Fat 0

	% Daily Value*
Total Fat 0g	0%
Saturated Fat 0g	0%
Cholesterol 0mg	0%
Sodium 390mg	15%
Total Carbohydrate 4g	1%
Dietary Fiber 2g	6%
Sugars 2g	
Protein 1g	

MEASUREMENT Complete.

26. ▨ feet $= \dfrac{1}{2}$ mile

27. ▨ ounces $= \dfrac{3}{4}$ pound

28. ▨ minutes $= 1\dfrac{2}{3}$ hours

29. ▨ quarts $= 5\dfrac{3}{8}$ gallons

ALGEBRA Evaluate each expression if $r = -\frac{1}{4}$, $s = \frac{2}{5}$, $t = \frac{8}{9}$, and $v = -\frac{2}{3}$.

30. rs

31. rt

32. stv

33. rtv

Find each product. Write in simplest form.

34. $\frac{1}{3} \cdot \left(-\frac{3}{8}\right) \cdot \frac{4}{5}$

35. $\frac{1}{2} \cdot \frac{2}{5} \cdot \frac{3}{4}$

36. $\left(-\frac{2}{5}\right) \cdot \frac{1}{6} \cdot \left(-\frac{5}{2}\right)$

37. $2\frac{2}{7} \cdot 1\frac{5}{9} \cdot 2\frac{2}{5}$

38. $3\frac{1}{3} \cdot 1\frac{1}{2} \cdot 5$

39. $10 \cdot 3.78 \cdot \frac{1}{5}$

40. $\frac{1}{5} \cdot 0.25$

41. $-\frac{2}{9} \cdot 0.\overline{3}$

42. $-\frac{7}{16} \cdot (-2.375)$

GEOGRAPHY For Exercises 43–45, refer to the table and the information below. Round answers to the nearest whole number.

There are about 57 million square miles of land on Earth covering seven continents.

43. What is the approximate land area of Europe?

44. What is the approximate land area of Asia?

45. Only about $\frac{3}{10}$ of Australia's land area is able to support agriculture. What fraction of the Earth's land is this?

Continent	Approximate Fraction of Earth's Landmass
Africa	$\frac{1}{5}$
Antarctica	$\frac{9}{100}$
Asia	$\frac{3}{10}$
Australia	$\frac{11}{200}$
Europe	$\frac{7}{100}$
North America	$\frac{33}{200}$
South America	$\frac{3}{25}$

ALGEBRA Evaluate each expression if $a = -1\frac{1}{5}$, $b = 2\frac{7}{9}$, $c = -2\frac{1}{4}$, and $d = 4\frac{1}{2}$. Express in simplest form.

46. abd^2

47. b^2c^2

48. $\frac{1}{2}a^2d$

49. $-3ac(-bd)$

50. **RESEARCH** Use the Internet or other resource to find a recipe for spaghetti sauce. Change the recipe to make $\frac{2}{3}$ of the amount. Then change the recipe to make $1\frac{1}{2}$ of the amount.

H.O.T. Problems

51. **FIND THE ERROR** Matt and Enrique are multiplying $2\frac{1}{2}$ and $3\frac{1}{4}$. Who is correct? Explain your reasoning.

$2\frac{1}{2} \cdot 3\frac{1}{4} = 2 \cdot 3 + \frac{1}{2} \cdot \frac{1}{4}$
$= 6 + \frac{1}{8}$
$= 6\frac{1}{8}$

Matt

$2\frac{1}{2} \cdot 3\frac{1}{4} = \frac{5}{2} \cdot \frac{13}{4}$
$= \frac{65}{8}$
$= 8\frac{1}{8}$

Enrique

52. OPEN ENDED Select two fractions whose product is greater than $\frac{1}{2}$ and less than 1. Use a number line to justify your answer.

53. CHALLENGE Find the missing fraction. $\frac{3}{4} \cdot \blacksquare = \frac{9}{14}$

54. WRITING IN MATH Explain why the product of $\frac{1}{2}$ and $\frac{7}{8}$ is less than $\frac{1}{2}$.

TEST PRACTICE

55. What number will make $\frac{3}{4} \cdot \frac{7}{8} = \frac{7}{8} \cdot n$ true?

A $\frac{4}{8}$

B $\frac{3}{4}$

C $\frac{10}{12}$

D $\frac{7}{8}$

56. Find the area of the triangle. Use the formula $A = \frac{1}{2}bh$.

$h = \frac{2}{3}$ in.

$b = 1\frac{1}{8}$ in.
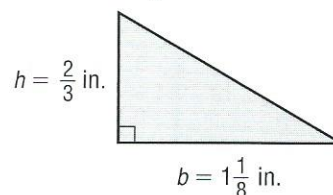
F $\frac{3}{4}$ in²   **H** $\frac{3}{8}$ in²

G $\frac{5}{8}$ in²   **J** $\frac{1}{6}$ in²

Spiral Review

Replace each ● with <, >, or = to make a true sentence. (Lesson 2-2)

57. $\frac{1}{2}$ ● $\frac{4}{7}$

58. $\frac{2}{7}$ ● $0.\overline{28}$

59. $-\frac{4}{9}$ ● $-0.\overline{4}$

60. HISTORY In 1864, Abraham Lincoln won the presidential election with about 0.55 of the popular vote. Write this as a fraction in simplest form. (Lesson 2-1)

61. GOLF After four rounds of golf, Lazaro's score was 5 under par or −5. Lazaro had improved his overall score during the fourth round by decreasing it by 6 strokes. Write and solve a subtraction equation to find Lazaro's score after the third round. (Lesson 1-9)

Write an equation to model the relationship between the quantities in each table. (Lesson 1-7)

62.

Servings, s	Total Calories, C
2	300
5	750
7	1,050
s	C

63.

Regular Price, p	Sale Price, s
$8	$6
$12	$9
$16	$12
p	s

GET READY for the Next Lesson

PREREQUISITE SKILL Divide. (Lesson 1-6)

64. $51 \div (-17)$ **65.** $-81 \div (-3)$ **66.** $-92 \div 4$ **67.** $-105 \div (-7)$