7-7 Surface Area of Prisms and Cylinders

Main IDEA

Find the lateral and total surface areas of prisms and cylinders.

Targeted TEKS 8.8 The student uses procedures to determine measures of three-dimensional figures. **(A) Find lateral and total surface area of prisms,** pyramids, **and cylinders using concrete models and nets** (two-dimensional models). **(C) Estimate measurements and use formulas to solve application problems involving lateral and total surface area** and volume.

NEW Vocabulary

lateral face
lateral surface area
total surface area

Vocabulary Link

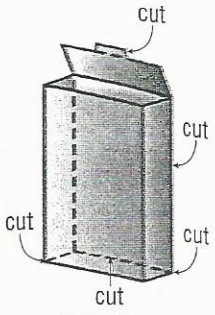
Lateral

Everyday Use situated on, directed toward, or coming from the side

Lateral face

Math Use a face of a solid that is not a base

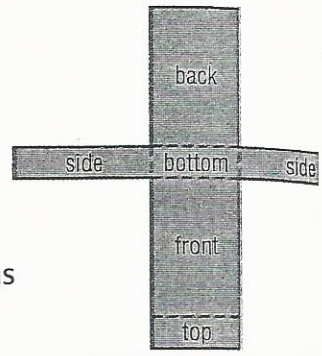
MINI Lab



STEP 1 Use an empty box with a tuck-in lid. Measure and record the height of the box and the perimeter of the top or bottom face.

STEP 2 Label the top, bottom, front, back, and side faces using a marker.

STEP 3 Open the lid and make 5 cuts as shown. Then open the box and lay it flat to form a net of the box. Measure and record the dimensions of each face.



1. Find the area of each face. Then find the sum of these areas.
2. Multiply the perimeter of a base by the height of the box. What does this product represent?
3. Add the product from Exercise 2 to the sum of the areas of the two bases.
4. Compare your answers from Exercises 1 and 3.

In the Mini Lab, you found the area of each surface, or face, of a box. A **lateral face** of a solid is any flat surface that is *not* a base. The **lateral surface area** of a solid is the sum of the areas of its lateral faces. The **total surface area** of a solid is the sum of the areas of all its surfaces.

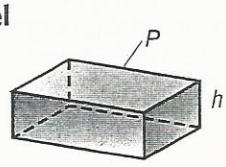
KEY CONCEPTS

Lateral Surface Area of a Prism

Words The lateral area L of a prism is the perimeter P of the base times the height h of the prism.

Symbols $L = Ph$

Model

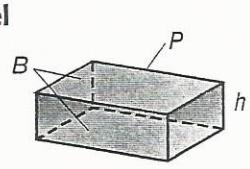


Total Surface Area of a Prism

Words The total surface area S of a prism is the lateral surface area L plus the area of the two bases $2B$.

Symbols $S = L + 2B$ or $S = Ph + 2B$

Model

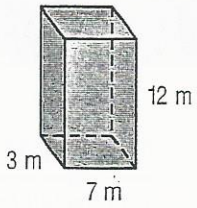


EXAMPLES Surface Areas of a Prism

STUDY TIP

Bases of Rectangular Prisms For the examples and exercises in this book, assume that the top and the bottom faces of a rectangular prism are its bases.

1 **Find the lateral and total surface areas of the rectangular prism.**



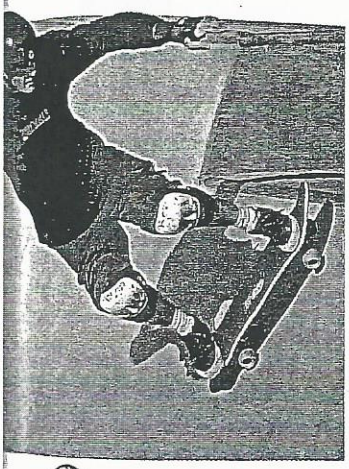
The bases of this prism are rectangles that are 3 meters wide and 7 meters long. Begin by finding the perimeter and area of one base.

Perimeter of Base	Area of Base
$P = 2\ell + 2w$	$B = \ell w$
$P = 2(7) + 2(3)$ or 20	$B = 7(3)$ or 21

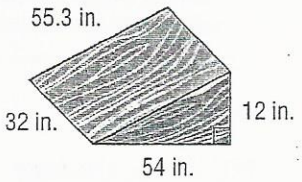
Use this information to find the lateral and total surface areas.

Lateral Surface Area	Total Surface Area
$L = Ph$	$S = L + 2B$
$L = 20(12)$ or 240	$S = 240 + 2(21)$ or 282

The lateral surface area is 240 square meters, and the total surface area of the prism is 282 square meters.



2 **SKATEBOARDING** **A wedge skateboarding ramp is built in the shape of a triangular prism. You plan to paint all surfaces of the ramp. Find the total surface area to be painted.**



Estimate $S = (60 + 50 + 10)(30) + 60(10)$ or 4,200 in^2.

The bases of the prism are triangles with side lengths of 12 inches, 54 inches, and 55.3 inches. Find the perimeter and area of one base.

Perimeter of Base	Area of Base
$P = 55.3 + 12 + 54$	$B = \frac{1}{2}bh$
$P = 121.3$	$B = \frac{1}{2}(54)(12)$ or 324

Use this information to find the total surface area.

$S = Ph + 2B$	Total surface area of prism
$S = 121.3(32) + 2(324)$	$P = 121.3$, $h = 32$, and $B = 324$.
$S = 4{,}529.6$	Simplify.

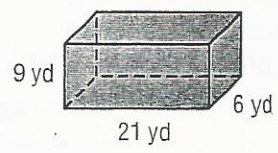
The surface area is 4,529.6 square inches. Compare to the estimate.

Real-World Link Other types of skateboarding ramps include angled boxes, lo-banks, quarterpipes, and micro halfpipes. Kits for building ramps can include isometric drawings of side and rear views.

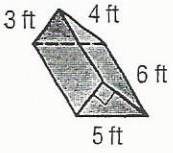
CHECK Your Progress

Find the lateral and total surface areas of each prism.

a.



b.



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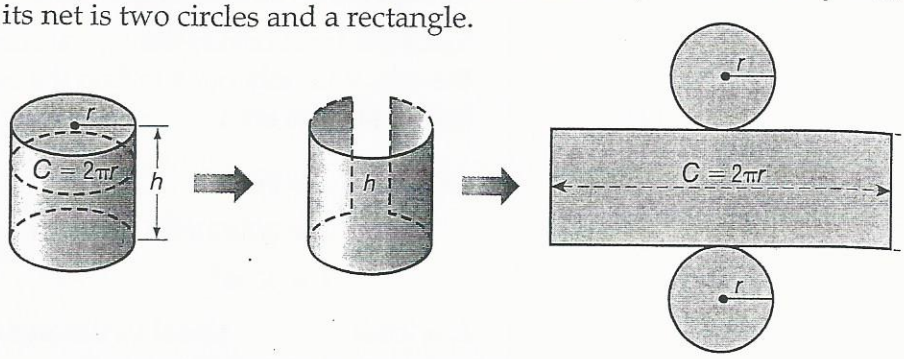
You can find the surface area of a cylinder by finding the area of its two bases and adding the area of the curved surface. The lateral a of a cylinder is the area of the curved surface. If you unfold a cylinde its net is two circles and a rectangle.



Model	Net	Area
2 circular bases	2 congruent circles with radius r	$2(\pi r^2)$ or $2\pi r^2$
1 curved surface	1 rectangle with width h and length $2\pi r$	$2\pi r \cdot h$ or $2\pi rh$

STUDY TIP

Cylinders The lateral and total surface areas of cylinders are found the same way as with prisms.

Prism: $L = Ph$
For cylinders, the base is a circle, so its perimeter is the circumference.

Prism: $S = L + 2B$
For cylinders, the base B is a circle with area πr^2.

Just as with prisms, you can use the measures of a cylinder to find the lateral and total surface areas of a cylinder.

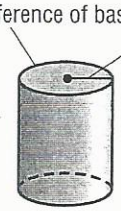
KEY CONCEPTS

Lateral Surface Area of a Cylinde

Words The lateral area L of a cylinder with height h and radius r is the circumference of the base times the height.

Symbols $L = 2\pi rh$

Model circumference of base = 2



Total Surface Area of a Cylinde

Words The surface area S of a cylinder with height h and radius r is the lateral area plus the area of the two bases.

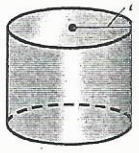
Symbols $S = L + 2\pi r^2$ or $S = 2\pi rh + 2\pi r^2$

Model

area of a base = πr^2

EXAMPLES Surface Areas of Cylinders

3 Find the lateral area and the surface area of the cylinder. Round to the nearest tenth.



Lateral Surface Area

$L = 2\pi rh$

$L = 2\pi(2)(3)$

$L \approx 37.7$

Total Surface Area

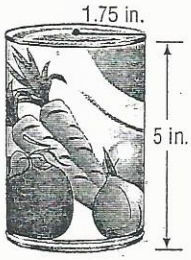
$S = L + 2\pi r^2$

$S = 37.7 + 2\pi(2)^2$

$S \approx 62.8$

The lateral area is about 37.7 square feet, and the surface area of th cylinder is about 62.8 square feet.

4 **LABELS** Find the area of the label on the can of vegetables shown at the right.



Since the label covers the lateral surface of the can, you only need to find the can's lateral surface area.

Estimate $L = 2\pi rh$

$L = 2(3)(2)(5)$ $\pi \approx 3, r = 1.75 \approx 2, h = 5$

$L \approx 60 \text{ in}^2$

$L = 2\pi rh$ Lateral surface area of cylinder

$L = 2\pi(1.75)(5)$ $r = 1.75, h = 5$

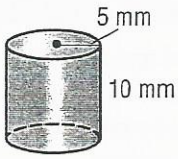
$L \approx 55.0$ Simplify.

The area of the label is about 55 square inches. Compare to the estimate.

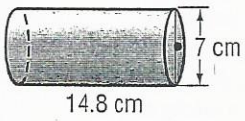
CHECK Your Progress

Find the lateral and total surface areas of each cylinder. Round to the nearest tenth.

c.



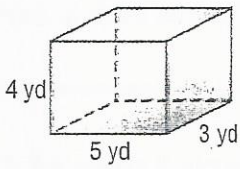
d.



CHECK Your Understanding

Examples 1, 2 (p. 387) **Find the lateral and total surface areas of each solid. Round to the nearest tenth if necessary.**

1.



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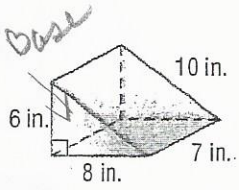
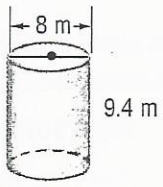
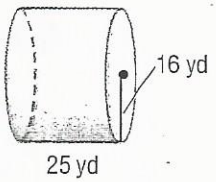


Example 3 (p. 388)

3.



4.



Example 4 (p. 389)

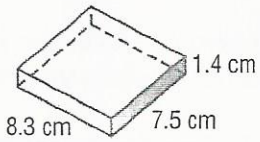
5. **CONTAINERS** Frozen orange juice often comes in cylindrical cardboard containers with metal lids. Find the area of the cardboard portion of the orange juice container shown.

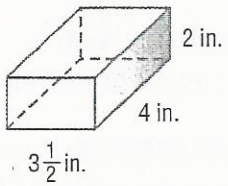
Exercises

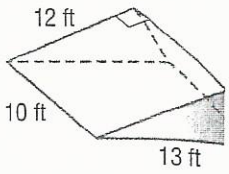
HOMEWORK HELP

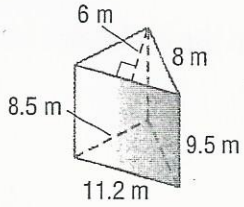
For Exercises	See Examples
6, 7	1
8, 9, 13	2
10, 11	3
12, 13	4

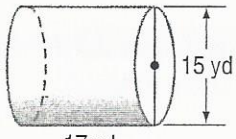
Find the lateral and total surface areas of each solid. Round to the neare tenth if necessary.

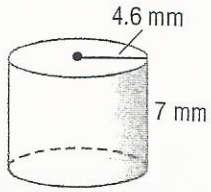
6. 

7. 

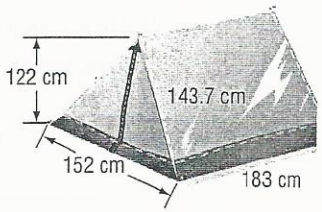
8. 

9. 

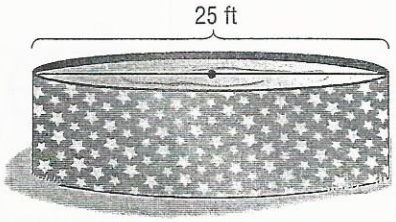
10. 

11. 

12. **CAMPING** A manufacturer makes nylon tents like the one shown. How much material is needed to make the tent?

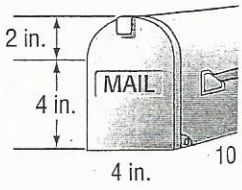


13. **POOL** A vinyl liner covers the inside walls and bottom of the swimming pool. Find the area of this liner to the nearest square foot.

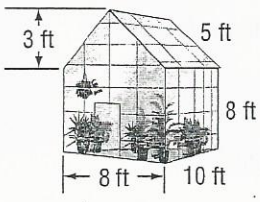


14. A rectangular prism has length 12 centimeters and width 4 centimeter its surface area is 467 square centimeters, what is the height of the pris

15. **MANUFACTURING** Find the amount of metal needed to construct the mailbox at the right to the nearest tenth of a square inch.



16. **GARDENING** The door of the greenhouse below has an area of 4.5 square feet. How many square feet of plastic are needed to cover the roof and sides of the greenhouse?



17. **PLUMBING** A hollow piece of a cylindrical pipe is shown. Find the total surface area of the pipe, including the interior.

EXTRA PRACTICE

See pages 713, 734.

Math Online

Self-Check Quiz at tx.msmath3.com

H.O.T. Problems

18. **REASONING** Determine whether the following statement is *true* or *false*. If *false*, give a counterexample.

If two rectangular prisms have the same volume, then they also have the same surface area.

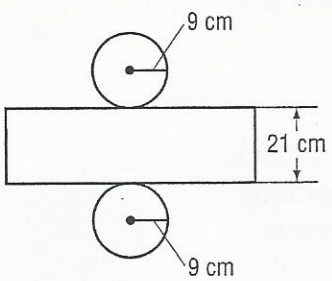
19. **CHALLENGE** Will the surface area of a cylinder increase more if you double the height or double the radius? Explain your reasoning.

20. **NUMBER SENSE** If you double the edge length of a cube, explain how this affects the surface area of the prism.

21. **WRITING IN MATH** Explain the difference between lateral area and surface area.

TEST PRACTICE

22. Fatima has a pattern for a pillow shown below.

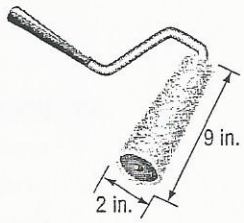


Which is closest to the total surface area of the pillow?

A 1,187.5 cm^2 C 1,442.0 cm^2

B 1,390.9 cm^2 D 1,696.5 cm^2

23. **GRIDDABLE** A paint roller like the one shown is used for painting large surfaces.



To the nearest tenth, how many square inches does a single rotation of the paint roller cover? Use 3.14 for π.

Spiral Review

Find the volume of each solid. Round to the nearest tenth if necessary. (Lesson 7-6)

24. rectangular pyramid: length, 14 m; width, 12 m; height, 7 m

25. cone: diameter, 22 cm; height, 24 cm

26. **HEALTH** The inside of a refrigerator in a medical laboratory measures 17 inches by 18 inches by 42 inches. You need at least 8 cubic feet to refrigerate some samples from the lab. Is the refrigerator large enough for the samples? Explain your reasoning. (Lesson 7-5)

GET READY for the Next Lesson

PREREQUISITE SKILL Multiply. (Lesson 2-3)

27. $\frac{1}{2} \cdot 2.8$ 28. $\frac{1}{2} \cdot 10 \cdot 23$ 29. $\frac{1}{2} \cdot 2.5 \cdot 16$ 30. $\frac{1}{2}\left(3\frac{1}{2}\right)(20)$