7-1 Circumference and Area of Circles

Main IDEA

Find the circumference and area of circles.

Reinforcement of TEKS 7.9 The student solves application problems involving estimation and measurement. **(A) Estimate measurements and solve application problems involving length (including circumference) and area of polygons and other shapes.** *Also addresses TEKS 8.10(A).*

NEW Vocabulary

circle
center
radius
diameter
circumference
pi

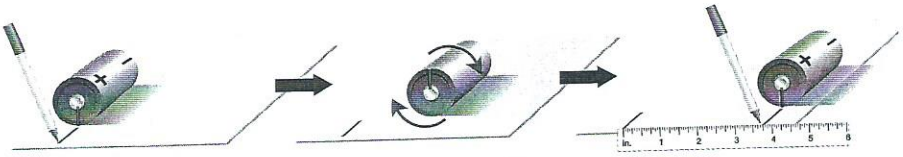
MINI Lab

STEP 1 Measure and record the distance d across the circular part of an object, such as a battery or a can, through its center.

STEP 2 Place the object on a piece of paper. Mark the point where the object touches the paper on both the object and on the paper.

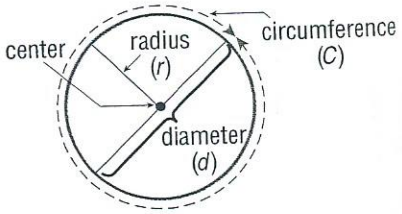
STEP 3 Carefully roll the object so that it makes one complete rotation. Then mark the paper again.

STEP 4 Finally, measure the distance C between the marks.



1. What distance does C represent?
2. Find the ratio $\frac{C}{d}$ for this object.
3. Repeat the steps above for at least two other circular objects and compare the ratios of C to d. What do you observe?
4. Graph the data you collected as ordered pairs, (d, C). Then describe the graph.

A **circle** is a set of points in a plane that are the same distance from a given point in the plane, called the **center**. The distance from the center to any point on the circle is called the **radius**. The distance across the circle through the center is its **diameter**. The distance around the circle is called the **circumference**.



The diameter of a circle is twice its radius or $d = 2r$.

STUDY TIP

Pi The numbers 3.14 and $\frac{22}{7}$ are often used as approximations for π.

The relationship you discovered in the Mini Lab is true for all circles. The ratio of the circumference of a circle to its diameter is always 3.1415926.... The Greek letter **π (pi)** represents this number.

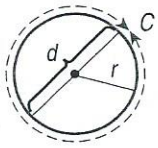
So, $\frac{C}{d} = \pi$. This can also be written as $C = \pi d$.

KEY CONCEPT — Circumference of a Circle

Words The circumference C of a circle is equal to its diameter d times π, or 2 times its radius r times π.

Symbols $C = \pi d$ or $C = 2\pi r$

Model



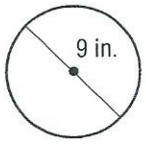
STUDY TIP

Calculating with π When evaluating expressions involving π, use the π key on a calculator to obtain the most accurate result. However, using 3.14 for π will result in a close approximation.

EXAMPLES Find the Circumferences of Circles

Find the circumference of each circle. Round to the nearest tenth.

1



$C = \pi d$ Circumference of a circle

$C = \pi \cdot 9$ Replace d with 9.

$C = 9\pi$ This is the *exact* circumference.

Use a calculator to find 9π. 9 × 2nd [π] ENTER 28.27433388

The circumference is about 28.3 inches.

2

$C = 2\pi r$ Circumference of a circle

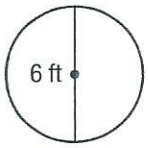
$C = 2 \cdot \pi \cdot 7.2$ Replace r with 7.2.

$C \approx 45.2$ Use a calculator.

The circumference is about 45.2 centimeters.

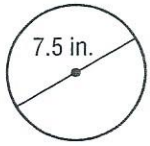
CHECK Your Progress

a.



b.

c.



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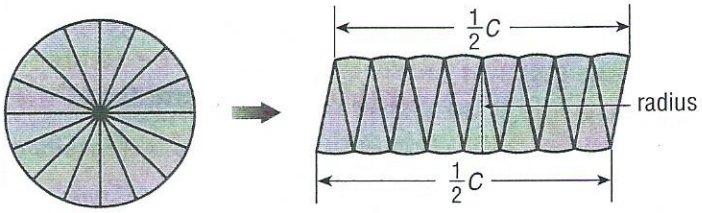
A circle can be separated into congruent wedge-like pieces. Then the pieces can be rearranged to form a figure that resembles a parallelogram.



Since the circle has an area that is relatively close to the area of the parallelogram-shaped figure, you can use the formula for the area of a parallelogram to find the formula for the area of a circle.

$A = bh$ Area of a parallelogram

$A = \left(\frac{1}{2} \cdot C\right)r$ The base of the parallelogram is one-half the circumference and the height is the radius.

$A = \left(\frac{1}{2} \cdot 2\pi r\right)r$ Replace C with $2\pi r$.

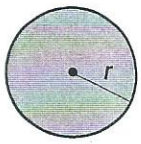
$A = \pi \cdot r \cdot r$ or πr^2 Simplify.

KEY CONCEPT — Area of a Circle

Words The area A of a circle is equal to π times the square of the radius r.

Symbols $A = \pi r^2$

Model



EXAMPLES Find the Areas of Circles

STUDY TIP

Estimation
To estimate the area of a circle, square the radius and then multiply by 3.

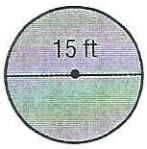
Find the area of each circle. Round to the nearest tenth.

3

$A = \pi r^2$	Area of a circle
$A = \pi \cdot 8^2$	Replace r with 8.
$A = \pi \cdot 64$	Evaluate 8^2.
$A \approx 201.1$	Use a calculator.

The area is about 201.1 square kilometers.

4



$A = \pi r^2$	Area of a circle
$A = \pi(7.5)^2$	Replace r with half of 15 or 7.5.
$A = \pi \cdot 56.25$	Evaluate 7.5^2.
$A \approx 176.7$	Use a calculator.

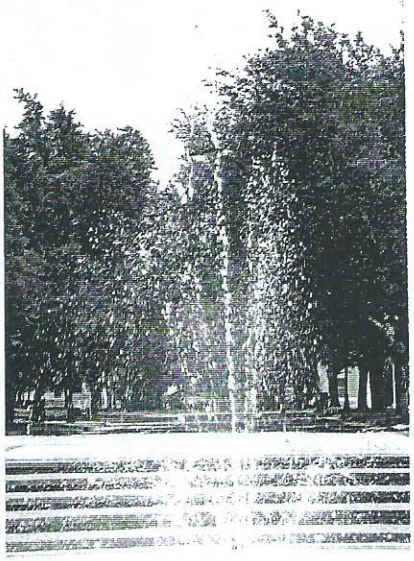
The area is about 176.7 square feet.

CHECK Your Progress

Find the area of each circle. Round to the nearest tenth.

d. The radius is 11 inches.

e. The diameter is 5 meters.



Real-World Link
The Jonsson Fountain at Austin College in Sherman, Texas, is a circular fountain that can be used as an amphitheater. As many as 500 people can be seated on tiered bricks circling the fountain.
Source: Austin College

Real-World EXAMPLE

5 **FOUNTAINS** **Refer to the information at the left. Suppose that you walk around the edge of the Jonsson Fountain and estimate its circumference to be 160 feet. Based on your estimate, what is the approximate diameter of the fountain?**

$C = \pi d$	Circumference of a circle
$160 = \pi d$	Replace C with 160.
$\frac{160}{\pi} = d$	Divide each side by π.
$50.9 \approx d$	Use a calculator.

The diameter of the fountain is about 51 feet.

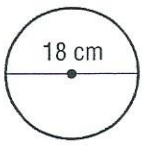
CHECK Your Progress

f. **HOME DECOR** A catalog states that a circular area rug covers 19.5 square feet. What is the diameter of the rug?

CHECK Your Understanding

Find the circumference of each circle. Round to the nearest tenth.

Examples 1, 2 (p. 353)

1. 

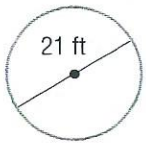
2. 

3. 

Find the area of each circle. Round to the nearest tenth.

Examples 3, 4 (p. 354)

4. 

5. 

6. 

Example 5 (p. 354)

7. **BRACELETS** When Cammie finished making a friendship bracelet, it was 7.9 inches long. What was the diameter of the bracelet?

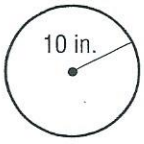
Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–11	1, 2
12–15	3, 4
16–19	5

Find the circumference of each circle. Round to the nearest tenth.

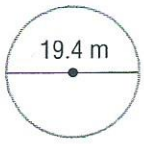
8. 

9. 

10. 

11. 

Find the area of each circle. Round to the nearest tenth.

12. 

13. 

14. 

15. 

16. **CARS** If the tires on a car each have a diameter of 25 inches, how far will the car travel in 100 rotations of its tires?

17. **FURNITURE** A circular table top has a radius of $2\frac{1}{4}$ feet. A decorative trim is placed along the outside edge of the table. How long is the trim?

18. **SAFETY** A light in a parking lot illuminates a circular area 15 meters across. What is the area of the parking lot covered by the light?

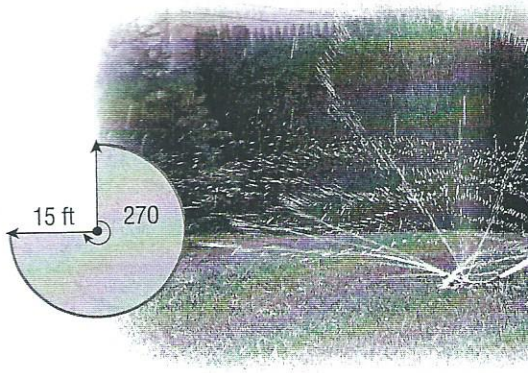
19. **ANIMALS** A California ground squirrel usually stays within 150 yards of its burrow. Find the area of a California ground squirrel's world.

Find the circumference and area of each circle. Round to the nearest tenth.

20. The radius is 3.5 centimeters.
21. The diameter is 8.6 kilometers.
22. The diameter is 9 inches.
23. The radius is 0.6 mile.

24. Find the diameter of a circle if its area is 706.9 square millimeters.

25. **LAWN CARE** The pattern of water distribution from a sprinkler is commonly a circle or part of a circle. A certain sprinkler is set to cover part of a circle measuring 270°. Find the area of the grass watered if the sprinkler reaches a distance of 15 feet.

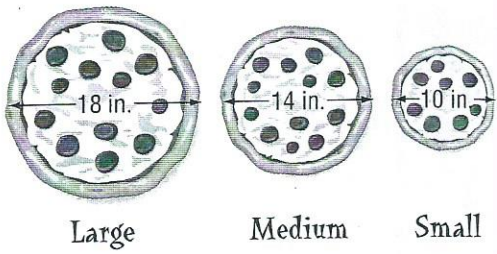


Another approximate value for π is $\frac{22}{7}$. Use this value to find the circumference and area of each circle.

26. The diameter is 7 feet.

27. The radius is $2\frac{1}{3}$ inches.

28. **PIZZA** The pizzeria has a special that offers one large, two medium, or three small pizzas for $12. Which offer is the best buy? Explain your reasoning.



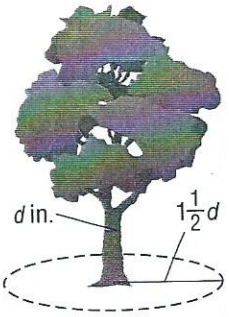
Real-World Link

Trees should be planted so that they have plenty of room to grow. The planting site should have an area of at least 2 to 3 times the diameter of the circle the spreading roots of the maturing tree are expected to occupy.

Source: www.forestry.uga.edu

29. **SPORTS** Three tennis balls are packaged one on top of the other in a can. Which measure is greater, the can's height or circumference? Explain.

30. **TREES** During a construction project, barriers are placed around trees. For each inch of trunk diameter, the protection zone should have a radius of $1\frac{1}{2}$ feet. Find the area of this zone for a tree with a trunk circumference of 63 inches.



31. **GRAPHIC ARTS** Michael is painting a sign for a new coffee shop. On the sign, he drew a circle with a radius of 2 feet. He then drew another circle with a radius 1.5 times larger. How much greater is the area of the larger circle?

EXTRA PRACTICE

See pages 711, 734.

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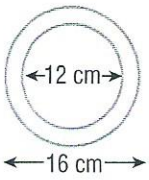
32. **FIND THE DATA** Refer to the Texas Data File on pages 16–19. Choose some data and write a real-world problem in which you would determine the area of a circle.

H.O.T. Problems

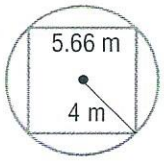
33. **OPEN ENDED** Draw and label a circle that has a circumference between 10 and 20 centimeters. Justify your answer.

34. **NUMBER SENSE** If the radius of a circle is halved, how will this affect its circumference and its area? What happens to the circumference and area if the radius is doubled or tripled? Explain your reasoning. (*Hint*: Find the circumference and area for each circle and organize the data in a table.)

CHALLENGE Find the area of each shaded region.

35. 

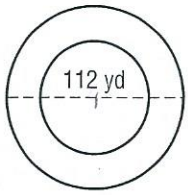
36. 

37. 

38. **WRITING IN MATH** Explain how to find the diameter of a circle if you know the measure of its area.

TEST PRACTICE

39. A circular track is shown below. Which is the closest to the distance traveled if you jog around the outside of the track twice?



A 351.9 yards
B 703.7 yards
C 1,407.4 yards
D 2,814.9 yards

40. The radius of the large plate is twice the radius of the small plate. If the area of the small plate is 19.6 square inches, what is the area of the large plate?

F 10 in^2
G 39.3 in^2
H 78.5 in^2
J 157 in^2

Spiral Review

For Exercises 41 and 42, use $\triangle ABC$ with vertices $A(-2, -2)$, $B(-1, 2)$, and $C(1, 0)$.

41. Graph $\triangle ABC$ and its image after it is translated 2 units right and 1 unit up. (Lesson 6-7)

42. What are the coordinates of $\triangle A'B'C'$ when $\triangle ABC$ is reflected over the x-axis? (Lesson 6-6)

43. **ART** At an auction in New York City, a 2.55-square inch portrait of George Washington sold for $1.2 million. About how much did the buyer pay per square inch for the portrait? (Lesson 4-1)

GET READY for the Next Lesson

44. **PREREQUISITE SKILL** The price of calculators has been decreasing. A calculator sold for $125 in 1995. A similar calculator sold for $89 in 2005. Use the *look for a pattern* strategy to determine the price of a similar calculator in 2025 if the price decrease continues at the same rate.