Dilations

NEW Vocabulary

dilation
center
enlargement
reduction

> **MINI Lab**
>
> The figure shown is drawn on 0.5-centimeter grid paper, so each square is 0.5-by-0.5 centimeter. Redraw the figure using squares that are 1-by-1 centimeter. Use point A as your starting point.
>
>
>
> 1. Measure and compare corresponding lengths on the original and new figure. Describe the relationship between these measurements. How does this relate to the change in grid-size?
>
> 2. **MAKE A CONJECTURE** What size squares should you use to create a version of the original figure with dimensions that are four times the corresponding lengths on the original? Explain.
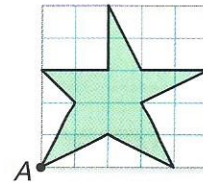
The image produced by enlarging or reducing a figure is called a **dilation**. A dilation image is similar to the original figure. This means that corresponding lengths on the two figures are proportional.

The **center** of the dilation is a fixed point used for measurement when altering the size of the figure. The ratio of a length on the image to a length on the original figure is the scale factor of the dilation.
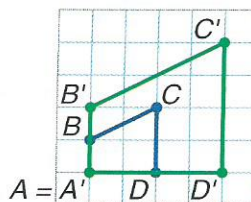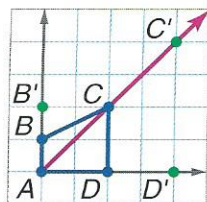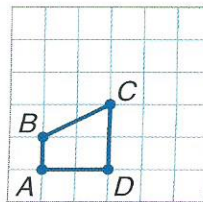
EXAMPLE   Draw a Dilation

① Copy polygon *ABCD* shown on graph paper. Then draw the image of the figure after a dilation with center *A* by a scale factor of 2.

Step 1   Draw ray *AB*, or $\overrightarrow{AB}$, extending it to the edges of the grid.

Step 2   Use a ruler to locate point *B'* on $\overrightarrow{AB}$ such that $AB' = 2(AB)$.

Step 3   Repeat Steps 1 and 2 for points *C'* and *D'*. Then draw polygon *A'B'C'D'* where $A = A'$.

STUDY TIP

Naming a Dilation
A dilated image is usually named using the same letters as the original figure, but with primes, as in polygon *A'B'C'D'*.

✓ **CHECK Your Progress**

a. Draw and label a large triangle *XYZ* on grid paper. Then draw the image of $\triangle XYZ$ after a dilation with center *X* and scale factor $\frac{1}{4}$.

In Example 1, if point A has coordinates $(0, 0)$, then the table below lists the coordinates of corresponding points on the original figure and its image.

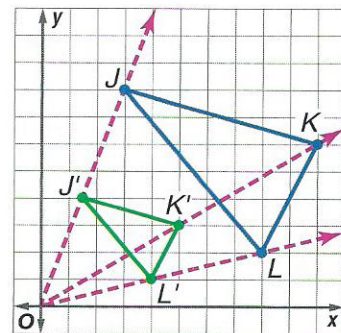| Original Coordinates	Relationship	Image Coordinates
$D(2, 0)$	$(2 \cdot 2, 0 \cdot 2)$	$D'(4, 0)$
$C(2, 2)$	$(2 \cdot 2, 2 \cdot 2)$	$C'(4, 4)$
$B(0, 1)$	$(0 \cdot 2, 1 \cdot 2)$	$B'(0, 2)$
$A(0, 0)$	$(0 \cdot 2, 0 \cdot 2)$	$A'(0, 0)$

To find the coordinates of the vertices of an image after a dilation with center $(0, 0)$, multiply the x- and y-coordinates by the scale factor.

EXAMPLE   Graph a Dilation

2 Graph $\triangle JKL$ with vertices $J(3, 8)$, $K(10, 6)$, and $L(8, 2)$. Then graph its image $\triangle J'K'L'$ after a dilation with a scale factor of $\frac{1}{2}$.

To find the vertices of the dilation, multiply each coordinate in the ordered pairs by $\frac{1}{2}$. Then graph both images on the same axes.

$$J(3, 8) \rightarrow \left(3 \cdot \frac{1}{2}, 8 \cdot \frac{1}{2}\right) \rightarrow J'\left(\frac{3}{2}, 4\right)$$
$$K(10, 6) \rightarrow \left(10 \cdot \frac{1}{2}, 6 \cdot \frac{1}{2}\right) \rightarrow K'(5, 3)$$
$$L(8, 2) \rightarrow \left(8 \cdot \frac{1}{2}, 2 \cdot \frac{1}{2}\right) \rightarrow L'(4, 1)$$

Check for Reasonableness
Draw lines through the origin and each of the vertices of the original figure. The vertices of the dilation should lie on those same lines.

CHECK Your Progress

Find the coordinates of the image of $\triangle JKL$ after a dilation with each scale factor. Then graph $\triangle JKL$ and $\triangle J'K'L'$.

b. scale factor: 3   **c.** scale factor: $\frac{1}{3}$

Examine the scale factors and the images produced after the dilations in Examples 1 and 2. These and other examples suggest the following.

- A dilation with a scale factor greater than 1 produces an **enlargement**, an image that is larger than the original figure.

- A dilation with a scale factor between 0 and 1 produces a **reduction**, an image that is smaller than the original figure.
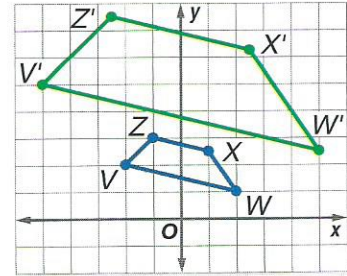
EXAMPLE **Find and Classify a Scale Factor**

3 Quadrilateral $V'Z'X'W'$ is a dilation of quadrilateral $VZXW$. Find the scale factor of the dilation, and classify it as an enlargement or a reduction.


Write a ratio of the x- or y-coordinate of one vertex of the dilation to the x- or y-coordinate of the corresponding vertex of the original figure. Use the y-coordinates of $V(-2, 2)$ and $V'(-5, 5)$.

$$\frac{y\text{-coordinate of point } V'}{y\text{-coordinate of point } V} = \frac{5}{2} \qquad \text{Verify by using other coordinates.}$$

The scale factor is $\frac{5}{2}$. Since $\frac{5}{2} > 1$, the dilation is an enlargement.

CHECK Your Progress

d. Triangle $A'B'C'$ is a dilation of $\triangle ABC$. Find the scale factor of the dilation, and classify it as an *enlargement* or a *reduction*.
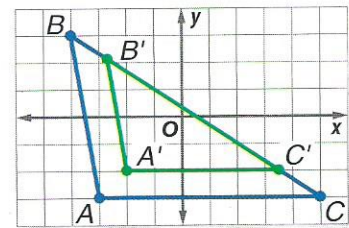
Before Dilation

After Dilation

Real-World EXAMPLE

4 **EYES** Carleta's optometrist dilates her pupils by a factor of $\frac{5}{3}$. If her pupil before dilation has a diameter of 5 millimeters, find the new diameter after her pupil is dilated.

Words	The size of the pupil after dilation	is $\frac{5}{3}$ the size of	the pupil before dilation.
Variable	Let a represent the size of the pupil after dilation.		
Equation	a	$= \frac{5}{3}$ $\cdot$	5

$a = \frac{5}{3}(5)$ Write the equation.

$a \approx 8.33$ Multiply.

Her pupil will be about 8.3 millimeters in diameter after dilation.

CHECK Your Progress

e. **COMPUTERS** Dante uses an image of his dog as the wallpaper on his computer desktop. The original image is 5 inches high and 7 inches wide. If his computer scales the image by a factor of $\frac{5}{4}$, what are the dimensions of the dilated image?
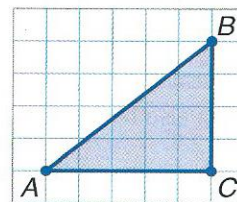
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Example 1
(p. 213)

Copy △ABC on graph paper. Then draw the image of the figure after the dilation with the given center and scale factor.

1. center: A, scale factor: $\frac{1}{2}$

2. center: C, scale factor: $\frac{3}{2}$

Example 2
(p. 214)

Triangle JKL has vertices $J(-4, 2)$, $K(-2, -4)$, and $L(3, 6)$. Find the vertices of △$J'K'L'$ after a dilation with the given scale factor. Then graph △JKL and △$J'K'L'$.

3. scale factor: 3

4. scale factor: $\frac{1}{4}$

Example 3
(p. 215)

5. On the graph, $\overline{A'B'}$ is a dilation of $\overline{AB}$. Find the scale factor of the dilation, and classify it as an *enlargement* or as a *reduction*.
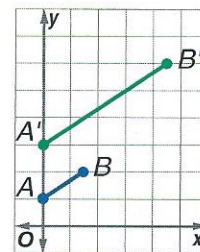
Example 4
(p. 215)

6. **GRAPHIC DESIGN** Simone designed a 6-inch by $7\frac{1}{2}$-inch logo for her school. The logo is to be reduced by a scale factor of $\frac{1}{3}$ and used to make face paintings. What are the dimensions of the dilated image?

Exercises

For Exercises	See Examples
7–10	1
11–14	2
15–18	3
19–20	4

HOMEWORK HELP

Copy each figure on graph paper. Then draw the image of the figure after the dilation with the given center and scale factor.
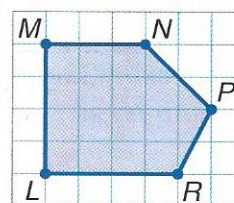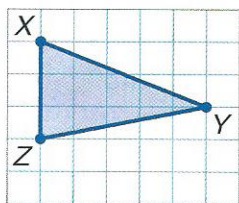
7. center: X, scale factor: $\frac{7}{3}$

8. center: Z, scale factor: $\frac{2}{3}$

9. center: L, scale factor: $\frac{3}{4}$

10. center: N, scale factor: 2

Find the vertices of polygon $H'J'K'L'$ after polygon $HJKL$ is dilated using the given scale factor. Then graph polygon $HJKL$ and polygon $H'J'K'L'$.

11. $H(-1, 3)$, $J(3, 2)$, $K(2, -3)$, $L(-2, -2)$; scale factor 2

12. $H(0, 2)$, $J(3, 1)$, $K(0, -4)$, $L(-2, -3)$; scale factor 3

13. $H(-6, 2)$, $J(4, 4)$, $K(7, -2)$, $L(-2, -4)$; scale factor $\frac{1}{2}$

14. $H(-8, 4)$, $J(6, 4)$, $K(6, -4)$, $L(-8, -4)$; scale factor $\frac{3}{4}$

On each graph, one figure is a dilation of the other. Find the scale factor of each dilation and classify it as an *enlargement* or as a *reduction*.

15.

16.

17.

18.
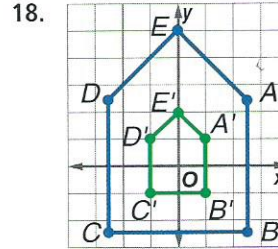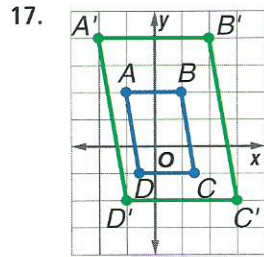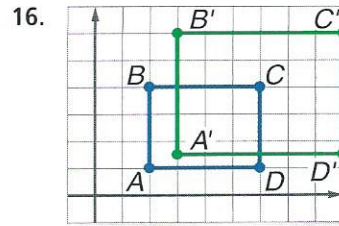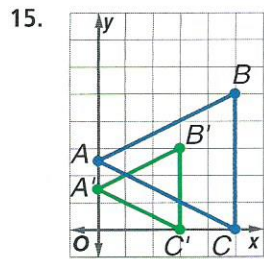
19. **PUBLISHING** To place a picture in his class newsletter, Joquin must reduce the picture by a scale factor of $\frac{3}{10}$. Find the dimensions of the reduced picture if the original is 15 centimeters wide and 10 centimeters high.

20. **PROJECTION** An overhead projector transforms the image on a transparency so that it is shown enlarged by a scale factor of 3.5 on a screen. If the original image is 3 inches long by 4 inches wide, find the dimensions of the projected image.

21. **BARN ART** Scott Hagan painted the Ohio bicentennial logo on one barn in each of Ohio's 88 counties. Each logo measured about 20 feet by 20 feet. Although Hagan drew each logo freehand, they are amazingly similar. If the original logo on which each painting was based measured 5 inches by 5 inches, what is the scale factor from the original logo to one of Hagan's paintings? Justify your answer.

22. **BUSINESS** A sporting goods store prints team mascots onto T-shirts. Javier selects the mascot shown. What is the scale factor of the dilation from the image he selects to the image as it appears on his T-shirt? Justify your answer.

2 in.

9 in.

23. **SCRAPBOOKING** Grace wants to fit the image of the certificate shown onto the top of an 8.5-inch by 11-inch page so that there is a 0.25-inch margin on its left and right sides. What is the scale factor of the dilation from the original certificate to its image on the page? Justify your answer.
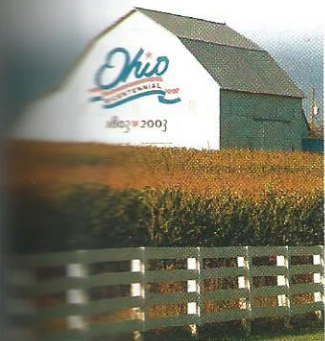
9 in.

8.5 in.

24. **OPEN ENDED** Graph a triangle and its image after a dilation with a scale factor greater than 1. Graph the resulting image after a dilation with a scale factor between 0 and 1. Predict the scale factor from the original to the final image. Explain your reasoning and verify your prediction.

25. **CHALLENGE** Describe the image of a figure after a dilation with a scale factor of −2.

26. **WRITING IN MATH** Write a general rule for finding the new coordinates of any ordered pair (x, y) after a dilation with a scale factor of k.
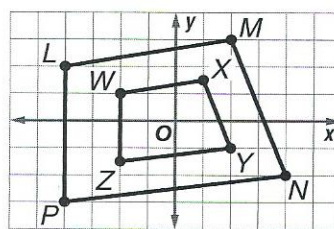
TEST PRACTICE

27. Square A is similar to square B.
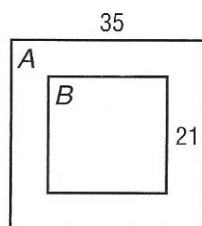
What scale factor was used to dilate square A to square B?

A $\frac{1}{7}$

B $\frac{3}{5}$

C $\frac{5}{3}$

D 7

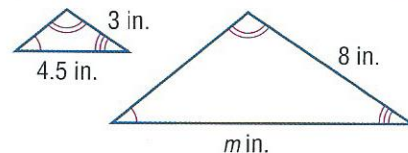28. Quadrilateral $LMNP$ was dilated to form quadrilateral $WXYZ$.

Which number best represents the scale factor used to change quadrilateral $LMNP$ into quadrilateral $WXYZ$?

F 3   H 2

G $\frac{1}{2}$ J $\frac{1}{3}$

Spiral Review

29. The triangles at the right are similar. Write and solve a proportion to find the missing measure. (Lesson 4-5)
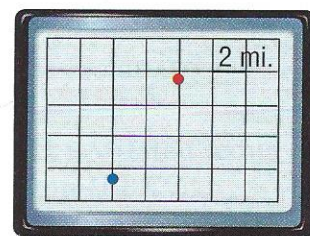

Solve each proportion. (Lesson 4-3)

30. $\frac{5}{4} = \frac{y}{12}$

31. $\frac{120}{b} = \frac{24}{60}$

32. $\frac{0.6}{5} = \frac{1.5}{n}$

33. **TECHNOLOGY** A backpacker uses her GPS (Global Positioning System) receiver to find how much farther she needs to go to get to her stopping point for the day. She is at the red dot on her GPS receiver screen, and the blue dot shows her destination. How much farther does she need to travel? (Lesson 3-7)

GET READY for the Next Lesson

PREREQUISITE SKILL Write a proportion and solve for x. (Lesson 4-3)

34. 3 cm is to 5 ft as x cm is to 9 ft

35. 4 in. is to 5 mi as 5 in. is to x mi