READING Math

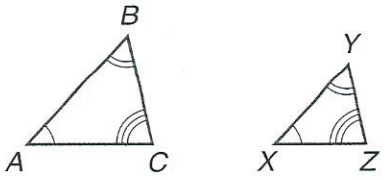
Congruence The symbol ≅ is read *is congruent to*. Arcs are used to show congruent angles.

The similar triangles in the Mini Lab suggest the following.

KEY CONCEPT — Similar Polygons

Words If two polygons are similar, then
- their corresponding angles are **congruent**, or have the same measure, and
- the measures of their corresponding sides are proportional.

Model

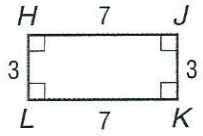


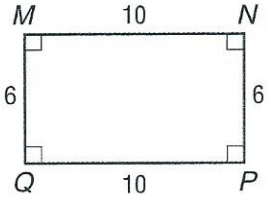
$\triangle ABC \sim \triangle XYZ$

Symbols $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$, and $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

EXAMPLE Identify Similar Polygons

1 **Determine whether rectangle *HJKL* is similar to rectangle *MNPQ*. Explain.**





First, check to see if corresponding angles are congruent.

Since the two polygons are rectangles, all of their angles are right angles. Therefore, all corresponding angles are congruent.

Next, check to see if corresponding sides are proportional.

$$\frac{HJ}{MN} = \frac{7}{10} \quad \frac{JK}{NP} = \frac{3}{6} \text{ or } \frac{1}{2} \quad \frac{KL}{PQ} = \frac{7}{10} \quad \frac{LH}{QM} = \frac{3}{6} \text{ or } \frac{1}{2}$$

Since $\frac{7}{10}$ and $\frac{1}{2}$ are not equivalent ratios, rectangle *HJKL* is *not* similar to rectangle *MNPQ*.

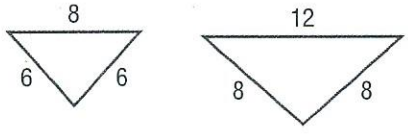
STUDY TIP

Common Error
Do not assume that two polygons are similar just because their corresponding angles are congruent. Their corresponding sides must also be proportional.

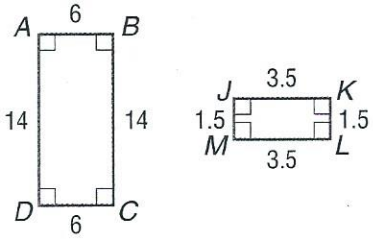
CHECK Your Progress

Determine whether these polygons are similar. Explain.

a.



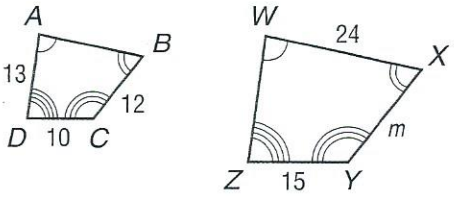
b.



The ratio of the lengths of two corresponding sides of two similar polygons is called the **scale factor**. You can use the scale factor of similar figures or a proportion to find missing measures.

EXAMPLE Find Missing Measures

2 **Given that polygon $WXYZ \sim$ polygon $ABCD$, find the missing measure.**



READING Math

Segment Measure The *measure* of $\overline{XY}$ is written as XY. It represents a number.

METHOD 1 **Write a proportion.**

The missing measure m is the length of $\overline{XY}$. Write a proportion that relates corresponding sides of the two polygons.

polygon $WXYZ \rightarrow \frac{XY}{BC} = \frac{YZ}{CD} \leftarrow$ polygon $WXYZ$
polygon $ABCD \rightarrow$ $\leftarrow$ polygon $ABCD$

$\frac{m}{12} = \frac{15}{10}$	$XY = m$, $BC = 12$, $YZ = 15$, and $CD = 10$.
$m \cdot 10 = 12 \cdot 15$	Find the cross products.
$10m = 180$	Multiply.
$m = 18$	Divide each side by 10.

METHOD 2 **Use the scale factor to write an equation.**

Find the scale factor from polygon $WXYZ$ to polygon $ABCD$ by finding the ratio of corresponding sides with known lengths.

scale factor: $\frac{YZ}{CD} = \frac{15}{10}$ or $\frac{3}{2}$ The scale factor is the constant of proportionality.

Words	A length on polygon $WXYZ$	is $\frac{3}{2}$ times as long as	a corresponding length on polygon $ABCD$.
Variable	Let m represent the measure of $\overline{XY}$.		
Equation	$m = \frac{3}{2} \cdot 12$		

$m = \frac{3}{2}(12)$	Write the equation.
$m = 18$	Multiply.

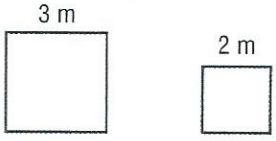
STUDY TIP

Scale Factor
In Example 2, the scale factor from polygon $ABCD$ to polygon $WXYZ$ is $\frac{2}{3}$, which means that a length on polygon $ABCD$ is $\frac{2}{3}$ as long as a length on polygon $WXYZ$.

CHOOSE Your Method **Find each missing measure above.**

c. WZ d. AB

Square A $\sim$ square B with a scale factor of 3:2. Notice the relationship between the scale factor and the ratio of their perimeters.



Square	Perimeter
A	12 m
B	8 m

perimeter of square A $\rightarrow$ $\frac{12}{8} = \frac{3}{2}$ or 3:2
perimeter of square B $\rightarrow$

This and other related examples suggest the following.

KEY CONCEPTS — Ratios of Similar Figures

Words If two figures are similar with a scale factor of $\frac{a}{b}$, then the perimeters of the figures have a ratio of $\frac{a}{b}$.

Model

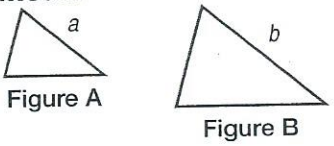


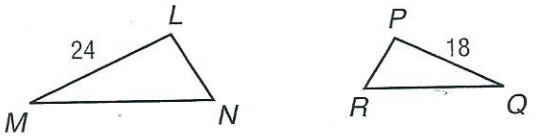
Figure A Figure B

TEST EXAMPLE

Test-Taking Tip

Similarity Statements In naming similar triangles, the order of the vertices indicates the corresponding parts. Read the similarity statement carefully to be sure that you compare corresponding parts.

3 Triangle LMN is similar to triangle PQR. If the perimeter of $\triangle LMN$ is 64 units, what is the perimeter of $\triangle PQR$?



A 108 units
B 96 units
C 48 units
D 36 units

Read the Test Item You know that the two triangles are similar, and you know the measures of two corresponding sides and the perimeter of $\triangle LMN$. You need to find the perimeter of $\triangle PQR$.

Solve the Test Item Triangle $LMN \sim$ triangle PQR with a scale factor of $\frac{24}{18}$ or $\frac{4}{3}$. The ratio of the perimeters of $\triangle LMN$ to $\triangle PQR$ is also $\frac{4}{3}$. Write and solve a proportion. Let x represent the perimeter of $\triangle PQR$.

perimeter of $\triangle LMN \rightarrow \frac{64}{x} = \frac{4}{3}$ ← perimeter of $\triangle PQR$ — Scale factor relating $\triangle LMN$ to $\triangle PQR$

$64 \cdot 3 = 4 \cdot x$ Find the cross products.

$192 = 4x$ Multiply.

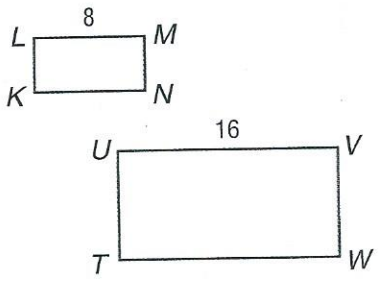
$\frac{192}{4} = \frac{4x}{4}$ Divide each side by 4.

$48 = x$ Simplify.

The answer is C.

CHECK Your Progress

e. Rectangle $KLMN$ is similar to rectangle $TUVW$. If the perimeter of rectangle $KLMN$ is 32 units, what is the perimeter of rectangle $TUVW$?



F 128 units
G 96 units
H 64 units
J 40 units

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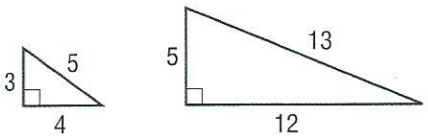
CHECK Your Understanding

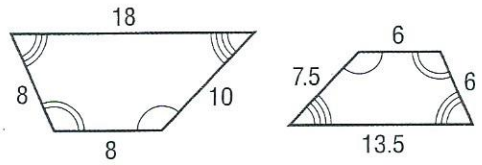
Example 1 (p. 206)

Determine whether each pair of polygons is similar. Explain.

1.

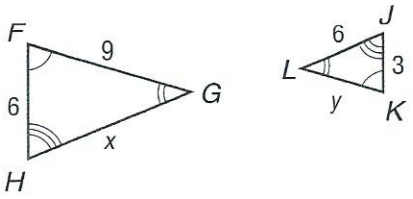


2.



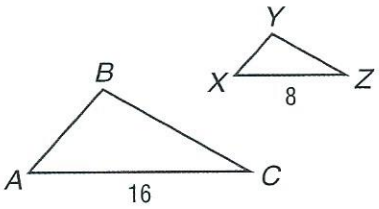
Example 2 (p. 207)

3. In the figure at the right, $\triangle FGH \sim \triangle KLJ$. Write and solve a proportion to find each missing side measure.



Example 3 (p. 208)

4. **TEST PRACTICE** $\triangle ABC$ is similar to $\triangle XYZ$. If the perimeter of $\triangle ABC$ is 40 units, what is the perimeter of $\triangle XYZ$?



A 10 units **C** 40 units

B 20 units **D** 80 units

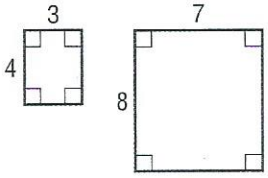
Exercises

HOMEWORK HELP

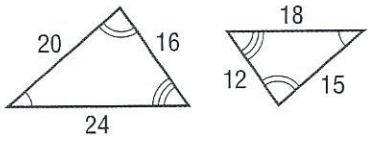
For Exercises	See Examples
5, 6	1
7–10	2
19, 20	3

Determine whether each pair of polygons is similar. Explain.

5.

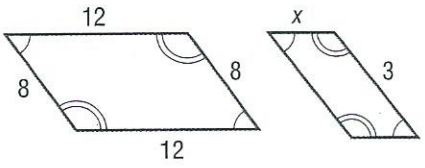


6.

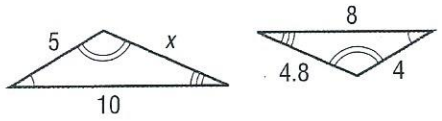


Each pair of polygons is similar. Write and solve a proportion to find each missing side measure.

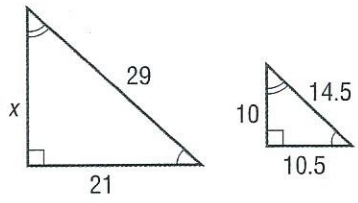
7.



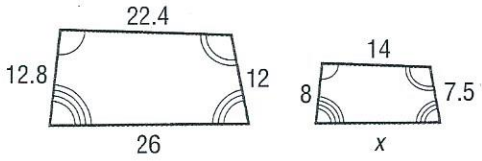
8.



9.

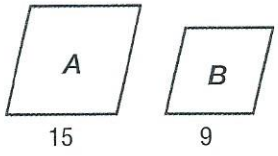


10.

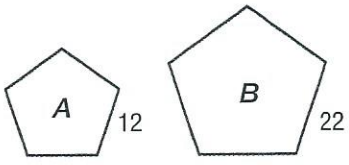


Each pair of polygons is similar. Determine the scale factor used to dilate figure A to figure B.

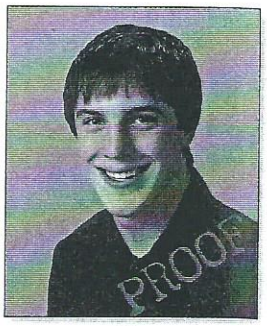
11.



12.



13. **YEARBOOK** The scale factor from the original proof at the right to the reduced picture for a yearbook will be 8:5. Find the dimensions of the pictures as they will appear in the yearbook.



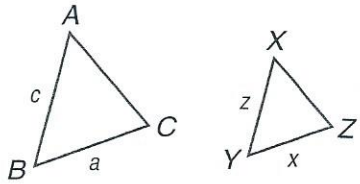
5 in.
4 in.

EXTRA PRACTICE
See pages 704, 731.
Math Online
Self-Check Quiz at tx.msmath3.com

14. **MOVIES** When projected onto a movie screen, the image from a film is 9 meters wide and 6.75 meters high. If the image from this same film is projected so that it appears 8 meters wide, what is the height of the projected image?

H.O.T. Problems

15. **CHALLENGE** *True* or *false*? If $\triangle ABC \sim \triangle XYZ$, then $\frac{a}{c} = \frac{x}{z}$. Justify your answer.



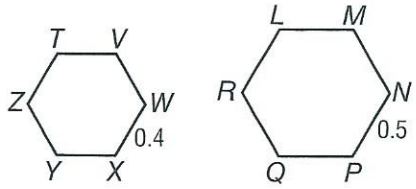
WRITING IN MATH **Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.**

16. Any two rectangles are similar.

17. Any two squares are similar.

TEST PRACTICE

18. Regular hexagon *LMNPQR* is similar to hexagon *TVWXYZ*.

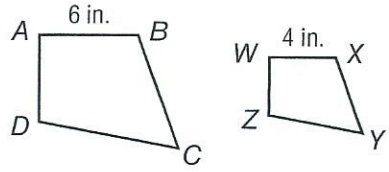


What scale factor was used to dilate regular hexagon *LMNPQR* to hexagon *TVWXYZ*?

A 0.1 **C** 0.8

B 0.75 **D** 1.25

19. Quadrilateral *ABCD* is similar to quadrilateral *WXYZ*.



If the area of quadrilateral *ABCD* is 54 square inches, what is the area of quadrilateral *WXYZ*?

F 13.5 inches^2 **H** 27 inches^2

G 24 inches^2 **J** 36 inches^2

Spiral Review

20. **ROCK CLIMBING** Grace is working her way up a climbing wall. Every 5 minutes she is able to climb 6 feet, but then loses her footing, slips back 1 foot, and decides to rest for 1 minute. If the rock wall is 30 feet tall, how long will it take her to reach the top? Use the *draw a diagram* strategy. (Lesson 4-4)

21. **BAKING** A recipe calls for 4 cups of flour for 64 cookies. How much flour is needed for 96 cookies? (Lesson 4-3)

GET READY for the Next Lesson

PREREQUISITE SKILL **Graph and connect each pair of ordered pairs.** (Lesson 3-6)

22. (−2.5, 1.5), (1.5, −3.5)

23. $\left(-2, -1\frac{1}{2}\right), \left(4, 3\frac{1}{2}\right)$

24. $\left(-2\frac{1}{3}, 1\right), \left(2, 3\frac{2}{3}\right)$