

# 4-3

# Solving Proportions

## Main IDEA

Use proportions to solve problems.



### Targeted TEKS 8.2

The student selects and uses appropriate operations to solve problems and justify solutions. **(D) Use multiplication by a constant factor (unit rate) to represent proportional relationships.**

**8.14** The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. **(A) Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics.** Also addresses TEKS 8.1(B), 8.3(B).

## NEW Vocabulary

equivalent ratios  
proportion  
cross products  
constant of proportionality

## STUDY TIP

**Cross Products** If the cross products of two ratios are equal, then the ratios form a proportion. If the cross products are *not* equal, the ratios do *not* form a proportion.

## GET READY for the Lesson

**NUTRITION** Part of the nutrition label from a granola bar is shown at the right.

- Write a ratio in simplest form that compares the number of Calories from fat to the total number of Calories.
- Suppose you plan to eat two such granola bars. Write a ratio comparing the number of Calories from fat to the total number of Calories.
- Is the number of Calories from fat proportional to the total number of Calories for one and two bars? Explain your reasoning.

## Nutrition Facts

Serving Size 1 Bar (28g)  
Servings Per Container 10

Amount Per Serving	
<b>Calories</b> 110	Calories from Fat 20
% Daily Value*	
<b>Total Fat</b> 2g	<b>3%</b>
Saturated Fat 0.5g	<b>2%</b>
<b>Cholesterol</b> 0mg	<b>0%</b>
<b>Sodium</b> 70mg	<b>3%</b>

In the example above, the ratios of Calories from fat to Calories for one or two granola bars are equal or **equivalent ratios** because they simplify to the same ratio,  $\frac{2}{11}$ . One way of expressing a proportional relationship like this is by writing a proportion.

$$\frac{20 \text{ Calories from fat}}{110 \text{ Calories}} = \frac{40 \text{ Calories from fat}}{220 \text{ Calories}}$$

## KEY CONCEPT

### Proportion

**Words** A **proportion** is an equation stating that two ratios or rates are equivalent.

**Examples**

**Numbers**

$$\frac{6}{8} = \frac{3}{4}$$

**Algebra**

$$\frac{a}{b} = \frac{c}{d}, b \neq 0, d \neq 0$$

Consider the following proportion.

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} \cdot \frac{1}{1} \cdot \frac{1}{1} = \frac{c}{d} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

$$ad = bc$$

Multiply each side by  $bd$  and divide out common factors.

Simplify.

The products  $ad$  and  $bc$  are called the **cross products** of this proportion. The cross products of any proportion are equal. You can use cross products to *solve proportions* in which one of the quantities is not known.

$$\begin{array}{l} \begin{array}{c} \cancel{6} \times \cancel{3} \\ \phantom{\cancel{6}} \times \phantom{\cancel{3}} \\ \cancel{8} \times \cancel{4} \end{array} \rightarrow 8 \cdot 3 = 24 \\ \phantom{\cancel{6}} \times \phantom{\cancel{3}} \\ \phantom{\cancel{8}} \times \phantom{\cancel{4}} \end{array} \rightarrow 6 \cdot 4 = 24$$

The cross products are equal.



## EXAMPLE Write and Solve a Proportion

- 1 **POOLS** After 2 hours, the water in a heated pool had risen  $7^{\circ}\text{C}$ . Write and solve a proportion to find the amount of time it will take at this rate for the temperature to rise an additional  $13^{\circ}\text{C}$ .

Write a proportion. Let  $t$  represent the time in hours.

$$\begin{array}{l} \text{temperature} \rightarrow \frac{7}{2} = \frac{13}{t} \leftarrow \text{temperature} \\ \text{time} \rightarrow \frac{7}{2} = \frac{13}{t} \leftarrow \text{time} \end{array}$$

$$\frac{7}{2} = \frac{13}{t} \quad \text{Write the proportion.}$$

$$7 \cdot t = 2 \cdot 13 \quad \text{Find the cross products.}$$

$$7t = 26 \quad \text{Multiply.}$$

$$\frac{7t}{7} = \frac{26}{7} \quad \text{Divide each side by 7.}$$

$$t \approx 3.7 \quad \text{Simplify.}$$

It will take about 3.7 hours for the temperature to rise an additional  $13^{\circ}\text{C}$ .

### CHECK Your Progress Solve each proportion.

a.  $\frac{x}{4} = \frac{9}{10}$

b.  $\frac{2}{34} = \frac{5}{y}$

c.  $\frac{7}{3} = \frac{n}{2.1}$

You can use ratios to make predictions in proportional situations.

### Real-World EXAMPLE

- 2 **BLOOD** A microscope slide shows 37 red blood cells and 23 blood cells that are not red blood cells. How many red blood cells would be expected in a sample of the same blood that has 925 blood cells?

$$\begin{array}{l} \text{red blood cells} \rightarrow \frac{37}{23 + 37} \text{ or } \frac{37}{60} \\ \text{total blood cells} \rightarrow \frac{37}{23 + 37} \text{ or } \frac{37}{60} \end{array}$$

Write and solve a proportion. Let  $r$  represent the number of red blood cells in the bigger sample.

$$\begin{array}{l} \text{red blood cells} \rightarrow \frac{37}{60} = \frac{r}{925} \leftarrow \text{red blood cells} \\ \text{total blood cells} \rightarrow \frac{37}{60} = \frac{r}{925} \leftarrow \text{total blood cells} \end{array}$$

$$37 \cdot 925 = 60 \cdot r \quad \text{Find the cross products.}$$

$$34,225 = 60r \quad \text{Multiply.}$$

$$\frac{34,225}{60} = \frac{60r}{60} \quad \text{Divide each side by 60.}$$

$$570.4 \approx r \quad \text{Simplify.}$$



#### Real-World Career . . .

**How Does a Medical Technologist Use Math?** A medical technologist uses proportional reasoning to analyze blood samples.



You can also use the constant ratio to write an equation expressing the relationship between two proportional quantities. The constant ratio is also called the **constant of proportionality**.

### EXAMPLE Write and Use an Equation

- 3 FUEL** Jaycee bought 8 gallons of gasoline for \$22.32. Write an equation relating the cost to the number of gallons of gasoline. How much would Jaycee pay for 11 gallons at this same rate? for 20 gallons?

Find the constant of proportionality between cost and gallons.

$$\frac{\text{cost in dollars}}{\text{gasoline in gallons}} = \frac{22.32}{8} \text{ or } 2.79 \quad \text{The cost is } \$2.79 \text{ per gallon.}$$

### STUDY TIP

**Checking Your Equation** You can check to see if the equation you wrote is accurate by testing the two known quantities.

$$\begin{aligned} c &= 2.79g \\ 22.32 &= 2.79(8) \\ 22.32 &= 22.32 \end{aligned}$$

**Words**

The cost is \$2.79 times the number of gallons.

**Variable**

Let  $c$  represent the cost.

Let  $g$  represent the number of gallons.

**Equation**

$$c = 2.79 \cdot g$$

Use this equation to find the cost for 11 and 20 gallons sold at the same rate.

$$\begin{aligned} c &= 2.79g && \leftarrow \text{Write the equation.} && \rightarrow c = 2.79g \\ c &= 2.79(11) && \leftarrow \text{Replace } g \text{ with the number of gallons.} && \rightarrow c = 2.79(20) \\ c &= 30.69 && \leftarrow \text{Multiply.} && \rightarrow c = 55.80 \end{aligned}$$

The cost for 11 gallons is \$30.69 and for 20 gallons is \$55.80.

### CHECK Your Progress

- e. **TYPING** Olivia typed 2 pages in 15 minutes. Write an equation relating the number of minutes  $m$  to the number of pages  $p$  typed. If she continues typing at this rate, how many minutes will it take her to type 10 pages? to type 25 pages?

## CHECK Your Understanding

**Example 1** Solve each proportion.

(p. 199)

1.  $\frac{1.5}{6} = \frac{10}{p}$

2.  $\frac{3.2}{9} = \frac{n}{36}$

3.  $\frac{41}{x} = \frac{5}{2}$

For Exercises 4 and 5, assume all situations are proportional.

**Example 2**

(p. 199)

4. **TEETH** For every 7 people who say they floss daily, there are 18 people who say they do not. Write and solve a proportion to determine out of 65 people how many you would expect to say they floss daily.

**Example 3**

(p. 200)

5. **TUTORING** Amanda earns \$28.50 tutoring for 3 hours. Write an equation relating her earnings  $m$  to the number of hours  $h$  she tutors. How much would Amanda earn tutoring for 2 hours? for 4.5 hours?



# Exercises

## HOMEWORK HELP

For Exercises	See Examples
6–15	1
16–19	2
20–25	3

Solve each proportion.

$$6. \frac{k}{7} = \frac{32}{56}$$

$$7. \frac{x}{13} = \frac{18}{39}$$

$$8. \frac{44}{p} = \frac{11}{5}$$

$$9. \frac{2}{w} = \frac{0.4}{0}$$

$$10. \frac{6}{25} = \frac{d}{30}$$

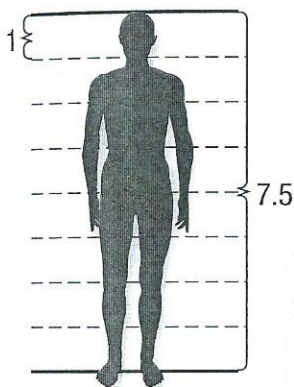
$$11. \frac{2.5}{6} = \frac{h}{9}$$

$$12. \frac{3.5}{8} = \frac{a}{3.2}$$

$$13. \frac{48}{9} =$$

For Exercises 14–21, assume all situations are proportional.

- COOKING** Evarado paid \$1.12 for a dozen eggs. Write and solve proportion to determine the ingredient cost of the 3 eggs Evarado needs for a recipe.
  - TRAVEL** A certain vehicle can travel 483 miles on 14 gallons of gas. Write and solve a proportion to determine how many gallons of gas this vehicle will need to travel 600 miles.
  - ILLNESS** For every person who actually has the flu, there are 6 persons who have flu-like symptoms resulting from a cold. If a doctor sees 40 patients, write and solve a proportion to determine how many of these you would expect to have a cold.
  - LIFE SCIENCE** For every left-handed person, there are about 4 right-handed persons. If there are 30 students in a class, write and solve a proportion to predict the number of students who are right-handed.
- **PEOPLE** For Exercises 18 and 19, use the following information.
- The head height to overall height ratio for an adult is given in the diagram at the left. Write and solve a proportion to predict the following measures.
  - 18. the height of an adult who has a head height of 9.6 inches
  - 19. the head height of an adult who is 64 inches tall
- PHOTOGRAPHY** It takes 2 minutes to print out 3 digital photos. Write an equation relating the number of photos  $n$  to the number of minutes  $m$ . At this rate, how long will it take to print 10 photos? 14 photos?
  - SPACE** A 20-pound object on Earth weighs  $3\frac{1}{3}$  pounds on the Moon. Write an equation relating the weight  $m$  of an object on the Moon to the weight  $a$  of the object on Earth. How much does an object weigh on the Moon if it weighs 96 pounds on Earth? 128 pounds on Earth?



### Real-World Link . . . . .

Although people vary in size and shape, in general, people do not vary in proportion.

Source: *Art Talk*

**MEASUREMENT** For Exercises 22–25, use the table to write an equation relating the two measures.

Customary System  
To Metric System



**H.O.T. Problems**

27. **LANDSCAPING** A 5-pound bag of grass seed covers 2,000 square feet. An opened bag has 3 pounds of seed remaining in it. Will this be enough to seed a 14-yard by 8-yard piece of land? Explain your reasoning.

28. **OPEN ENDED** List two other amounts of cinnamon and sugar, one larger and one smaller, that are proportional to  $1\frac{1}{2}$  tablespoons of cinnamon for every 3 tablespoons of sugar. Justify your answers.

**CHALLENGE** Solve each equation.

29.  $\frac{2}{3} = \frac{18}{x+5}$

30.  $\frac{x-4}{10} = \frac{7}{5}$

31.  $\frac{4.5}{17-x} = \frac{3}{8}$

32. **WRITING IN MATH** Explain why it might be easier to write an equation to represent a proportional relationship rather than using a proportion.

**TEST PRACTICE**

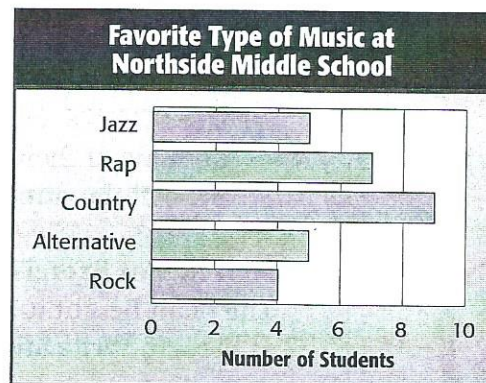
33. Michael paid \$24 for 3 previously-viewed DVDs at Play-It-Again Movies. Which equation can he use to find the cost  $c$  of purchasing 12 previously-viewed DVDs from this same store?

- A  $c = 12 \cdot 24$       C  $c = 12 \cdot 8$
- B  $c = 24 \cdot 4$       D  $c = 72 \cdot 36$

34. An amusement park line is moving about 4 feet every 15 minutes. At this rate, approximately how long will it take for a person at the back of the 50-foot line to reach the front of the line?

- F 1 hour
- G 3 hours
- H 5 hours
- J 13 hours

35. The graph shows the results of a survey of 30 Northside students.



Which proportion can be used to find  $n$ , the number preferring country music out of 440 Northside students?

- A  $\frac{30}{9} = \frac{n}{440}$       C  $\frac{n}{9} = \frac{30}{400}$
- B  $\frac{440}{n} = \frac{9}{30}$       D  $\frac{9}{30} = \frac{n}{440}$

**Spiral Review**

36. **MONEY** Cassie deposits \$40 in a savings account. The money earns \$1.40 per month in simple interest, and she makes no further deposits. Is her account balance proportional to the number of months since her initial deposit? (Lesson 4-2)

37. **SHOPPING** Which is the better buy: 1 pound 4 ounces of cheese for \$4.99 or 2 pounds 6 ounces for \$9.75? Explain your reasoning. (Lesson 4-1)

**GET READY for the Next Lesson**

38. **PREREQUISITE SKILL** Jacquelyn pays \$8 for fair admission but then must pay \$0.75 for each ride. If she rides five rides, what is the total cost at the fair? (Lesson 1-1)