Estimating Square Roots

## Main IDEA

Estimate square roots.

Targeted TEKS
8.1 The student understands that different forms of numbers are appropriate for different situations. **(C)** Approximate (mentally and with calculators) the value of irrational numbers as they arise from problem situations (such as π, $\sqrt{2}$).

▶ MINI Lab

STEP 1 On dot paper, draw and cut out a square like the one at the right. The area of section A is $\frac{1}{2}(2 \cdot 2)$ or 2 square units. So, the shaded square has an area of 8 square units.
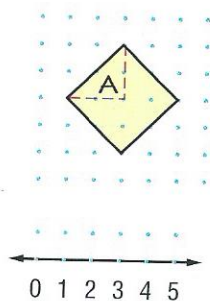
STEP 2 Draw a number line on your dot paper so that 1 unit equals the distance between dots.

0 1 2 3 4 5

1. Place your square on the number line. Between what two consecutive whole numbers is $\sqrt{8}$, the side length of the square, located?

2. Between what two perfect squares is 8 located?

3. Estimate the length of a side of the square. Verify your estimate by using a calculator to compute the value of $\sqrt{8}$.

In the Mini Lab, $\sqrt{8}$ is not a whole number since 8 is not a perfect square.

The number line shows that $\sqrt{8}$ is between 2 and 3. Since 8 is closer to 9 than 4, the best whole number estimate for $\sqrt{8}$ is 3.
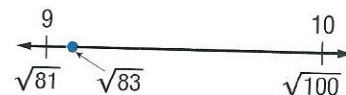
2   3
$\sqrt{4}$ $\sqrt{8}$ $\sqrt{9}$

EXAMPLES   Estimate Square Roots

1 **Estimate $\sqrt{83}$ to the nearest whole number.**

- The first perfect square less than 83 is 81. $\sqrt{81} = 9$
- The first perfect square greater than 83 is 100. $\sqrt{100} = 10$

Plot each square root on a number line. Then plot $\sqrt{83}$.

9   10
$\sqrt{81}$ $\sqrt{83}$ $\sqrt{100}$

READING Math

Inequalities
$81 < 83 < 100$ is read *81 is less than 83 is less than 100* or *83 is between 81 and 100.*

$$81 < 83 < 100 \quad \text{Write an inequality.}$$
$$9^2 < 83 < 10^2 \quad 81 = 9^2 \text{ and } 100 = 10^2$$
$$\sqrt{9^2} < \sqrt{83} < \sqrt{10^2} \quad \text{Find the square root of each number.}$$
$$9 < \sqrt{83} < 10 \quad \text{Simplify.}$$

So, $\sqrt{83}$ is between 9 and 10. Since $\sqrt{83}$ is closer to $\sqrt{81}$ than $\sqrt{100}$, the best whole number estimate for $\sqrt{83}$ is 9.

2 Estimate $\sqrt{23.5}$ to the nearest whole number.

- The first perfect square less than 23.5 is 16. $\sqrt{16} = 4$
- The first perfect square greater than 23.5 is 25. $\sqrt{25} = 5$

$16 < 23.5 < 25$	Write an inequality.
$4^2 < 23.5 < 5^2$	$16 = 4^2$ and $25 = 5^2$
$\sqrt{4^2} < \sqrt{23.5} < \sqrt{5^2}$	Find the square root of each number.
$4 < \sqrt{23.5} < 5$	Simplify.

So, $\sqrt{23.5}$ is between 4 and 5. Since 23.5 is closer to 25 than 16, the best whole number estimate for $\sqrt{23.5}$ is 5.
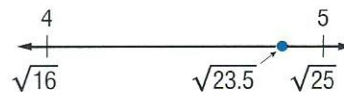
✔ **CHECK Your Progress**   Estimate to the nearest whole number.

a. $\sqrt{35}$ b. $\sqrt{44.8}$ c. $\sqrt{170}$

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Real-World EXAMPLE

3 **ART** The Parthenon is an example of a *golden rectangle*. The length of the longer side divided by the length of the shorter side is equal to $\frac{1 + \sqrt{5}}{2}$. Estimate this value.

2 units

$(1 + \sqrt{5})$ units

First estimate the value of $\sqrt{5}$.

$4 < 5 < 9$	4 and 9 are the closest perfect squares.
$2^2 < 5 < 3^2$	$4 = 2^2$ and $9 = 3^2$
$\sqrt{2^2} < \sqrt{5} < \sqrt{3^2}$	Find the square root of each number.
$2 < \sqrt{5} < 3$	Simplify.

Since 5 is closer to 4 than 9, the best whole number estimate for $\sqrt{5}$ is 2. Use this value to evaluate the expression.

$$\frac{1 + \sqrt{5}}{2} \approx \frac{1 + 2}{2} \text{ or } 1.5$$

✔ **CHECK Your Progress**

d. **BASEBALL** In Little League, the bases are squares with sides of 14 inches. The expression $\sqrt{(s^2 + s^2)}$ represents the distance *across* a square of side length s. Estimate the distance across a base to the nearest inch.
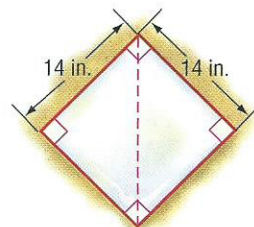
14 in.   14 in.

Examples 1, 2
(pp. 148–149)

Estimate to the nearest whole number.

1. $\sqrt{28}$
2. $\sqrt{60}$
3. $\sqrt{135}$
4. $\sqrt{13.5}$
5. $\sqrt{38.7}$
6. $\sqrt{79.2}$

Example 3
(p. 149)

7. **SCIENCE** The number of swings back and forth of a pendulum of length L, in inches, each minute is $\frac{375}{\sqrt{L}}$. About how many swings will a 40-inch pendulum make each minute?

Exercises

HOMEWORK HELP

For Exercises	See Examples
8–15	1, 2
16, 17	3

Estimate to the nearest whole number.

8. $\sqrt{44}$
9. $\sqrt{23}$
10. $\sqrt{125}$
11. $\sqrt{197}$
12. $\sqrt{15.6}$
13. $\sqrt{23.5}$
14. $\sqrt{85.1}$
15. $\sqrt{38.4}$

16. **GEOMETRY** The radius of a circle with area A is approximately $\sqrt{\frac{A}{3}}$. If a pizza has an area of 78 square inches, estimate its radius.

17. **CAVES** The formula $t = \frac{\sqrt{h}}{4}$ represents the time t in seconds that it takes an object to fall from a height of h feet. Suppose a rock falls from a 200-feet high cave ceiling. Estimate how long will it take to reach the ground.

Estimate to the nearest whole number.

18. $\sqrt{5\frac{1}{5}}$
9. $\sqrt{21\frac{7}{10}}$
20. $\sqrt{17\frac{3}{4}}$

Order from least to greatest.

21. $7, 9, \sqrt{50}, \sqrt{85}$
22. $\sqrt{91}, 7, 5, \sqrt{38}$
23. $\sqrt{62}, 6, \sqrt{34}, 8$

ALGEBRA Estimate the solution of each equation to the nearest integer.

24. $y^2 = 55$
25. $d^2 = 95$
26. $p^2 = 6.8$

27. **FIND THE DATA** Refer to the Texas Data File on pages 16–19. Choose some data and write a real-world problem in which you would estimate a square root.

EXTRA PRACTICE

See pages 701, 730.

Math Online

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28. **GEOMETRY** Egyptian mathematician Heron created the formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ to find the area A of a triangle. In this formula, a, b, and c are the measures of the sides, and s is one-half of the perimeter. Use this formula to estimate the area of the triangle at the right.

4 cm
6 cm
8 cm
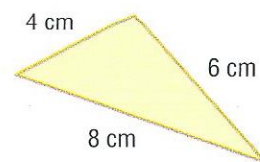
29. **NUMBER SENSE** Without a calculator, determine which is greater, $\sqrt{94}$ or 10. Explain your reasoning.

30. OPEN ENDED Find two numbers that have square roots between 7 and 8. One number should have a square root closer to 7, and the other number should have a square root closer to 8. Justify your answer.

31. FIND THE ERROR Josephina and Dario are estimating $\sqrt{50}$. Who is correct? Explain your reasoning.

$\sqrt{50} \approx 7$

$\sqrt{50} \approx 25$

Josephina

Dario

32. CHALLENGE If $x^3 = y$, then x is the cube root of y. Explain how to estimate the cube root of 30. Find the cube root of 30 to the nearest whole number.

33. WRITING IN MATH Explain how to graph $\sqrt{78}$ on a number line.

TEST PRACTICE

34. Ms. Michaels wrote four numbers on the board and asked Christine to circle the number closest to 7. Which number should Christine choose?

- **A** $\sqrt{3}$
- **B** $\sqrt{15}$
- **C** $\sqrt{45}$
- **D** $\sqrt{60}$

35. Point N on the number line best represents which square root?

$$N$$

9 10 11 12 13
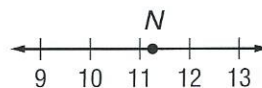
- **F** $\sqrt{140}$
- **G** $\sqrt{121}$
- **H** $\sqrt{116}$
- **J** $\sqrt{126}$

Spiral Review

36. ALGEBRA Find a number that, when squared, equals 8,100. (Lesson 3-1)

37. GEOGRAPHY The Great Lakes cover about 94,000 square miles. Write this number in scientific notation. (Lesson 2-10)

Multiply or divide. (Lesson 1-6)

38. $(-5)(-13)$ **39.** $(-2)(5)(7)$ **40.** $72 \div (-2)$ **41.** $-80 \div (-16)$

GET READY for the Next Lesson

42. PREREQUISITE SKILL To attend a field trip to an art museum, each student will have to pay $6.50 for transportation and $10.00 for admission and lunch. Find the total amount of money to be collected for a class of 240 students. (Lesson 1-1)