

7-8

Surface Area of Pyramids

Main IDEA

Find the lateral and total surface areas of pyramids.

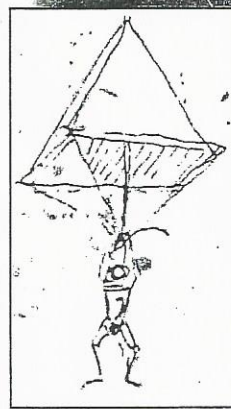


Targeted TEKS 8.8
The student uses procedures to determine measures of three-dimensional figures. (A) Find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two-dimensional models). (C) Estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume. Also addresses TEKS 8.10(B).

GET READY for the Lesson

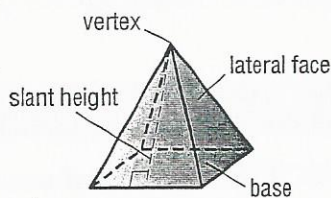
HISTORY In 1485, Leonardo Da Vinci sketched a pyramid-shaped parachute in the margin of his notebook. In June 2000, using a parachute created with tools and materials available in medieval times, Adrian Nicholas proved Da Vinci's design worked by descending 7,000 feet.

1. How many cloth faces does this pyramid have? What shape are they?
2. How could you find the total area of the material used for the parachute?

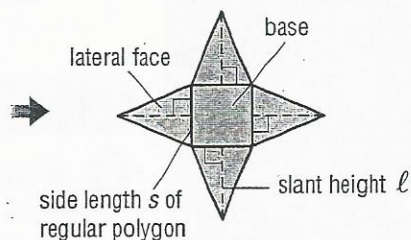


A **regular pyramid** is a pyramid whose base is a regular polygon. The lateral faces of a regular pyramid are congruent isosceles triangles, which intersect at the pyramid's vertex. The altitude or height of each lateral face is called the **slant height** of the pyramid.

Model of Regular Pyramid



Net of Regular Pyramid



STUDY TIP

Look Back

You can review **area of triangles** on page 693.

To find the lateral area L of a regular pyramid, look at its net. The lateral area of a pyramid is the sum of the areas of its lateral faces, which are all triangles.

The net of a square pyramid is a square and four triangles as shown above.

$$L = 4\left(\frac{1}{2}sl\right) \quad \text{Area of the lateral faces}$$

$$L = \frac{1}{2}(4s)\ell \quad \text{Commutative Property of Multiplication}$$

$$L = \frac{1}{2}P\ell \quad \text{The perimeter of the base } P \text{ is } 4s.$$

The total surface area of a regular pyramid is the lateral surface area plus the area of the base.

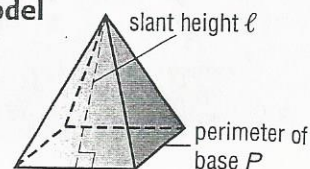
KEY CONCEPTS

Lateral Surface Area of a Pyramid

Words The lateral surface area L of a regular pyramid is half the perimeter P of the base times the slant height ℓ .

Symbols $L = \frac{1}{2}P\ell$

Model

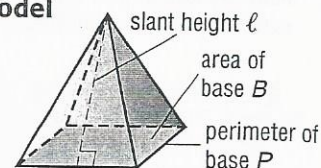


Total Surface Area of a Pyramid

Words The total surface area S of a regular pyramid is the lateral area L plus the area of the base B .

Symbols $S = L + B$ or $S = \frac{1}{2}P\ell + B$

Model



EXAMPLE Surface Areas of a Pyramid

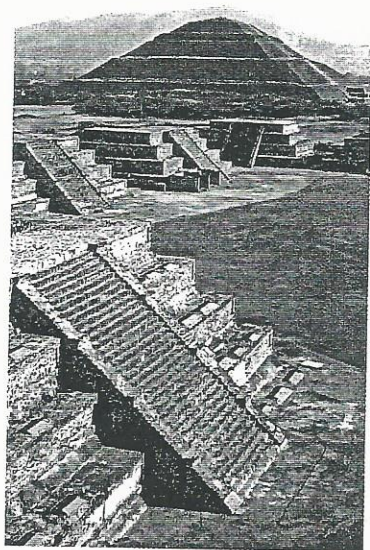
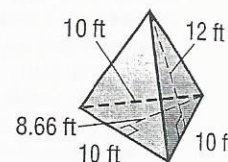
- 1 Find the lateral and total surface areas of the triangular pyramid.

$$L = \frac{1}{2}P\ell \qquad S = L + B$$

$$L = \frac{1}{2} \cdot 30 \cdot 12 \qquad S = 180 + 43.3 \qquad B = \frac{1}{2} \cdot 10 \cdot 8.66$$

$$L = 180 \qquad S = 223.3$$

The lateral and total surface areas are 180 and 223.3 square feet.



Real-World Link

The Pyramid of the Sun in Teotihuacán, Mexico, was built in the second century, A.D. It is about 71 meters tall, and its square base has side lengths of 223.5 meters.

Source: infoplease.com

CHECK Your Progress

- a. Find the lateral and total surface areas of a pyramid with a slant height of 18 meters and a square base with 11-meter sides.

Real-World EXAMPLE

- 2 **ARCHITECTURE** Use the information at the left to find the lateral surface area of the Pyramid of the Sun if it has a slant height of 132.5 meters.

$$L = \frac{1}{2}P\ell \qquad \text{Lateral surface area of a pyramid}$$

$$L = \frac{1}{2} \cdot 894 \cdot 132.5 \qquad P = 223.5(4) \text{ or } 894 \text{ and } \ell = 132.5$$

$$L = 59,227.5 \qquad \text{Simplify.}$$

The lateral area of the pyramid is 59,227.5 square meters.

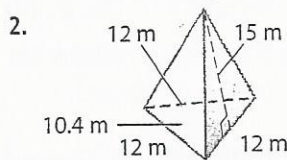
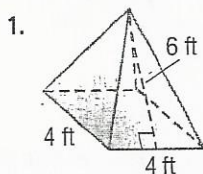
CHECK Your Progress

- b. **AWARDS** A music award is a square pyramid with a 6-inch long base and a 13-inch slant height. Find the award's total surface area.

Online Personal Tutor at tx.msmath3.com

CHECK Your Understanding

Example 1 (p. 394) Find the lateral and total surface areas of each regular pyramid. Round to the nearest tenth if necessary.



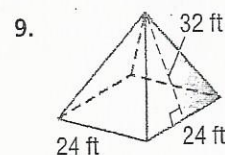
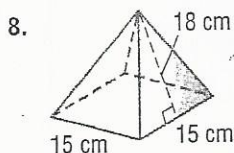
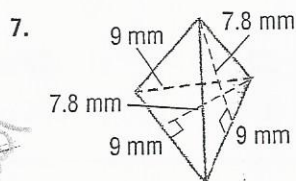
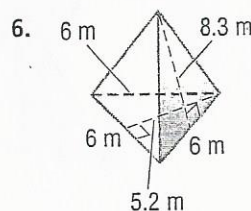
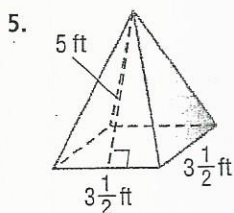
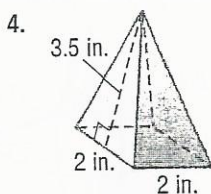
Example 2 (p. 394) 3. **HISTORY** Refer to the lesson opener. Each face of the parachute has a base of about 12 yards and a height of about 17 yards. Find the amount of material needed to make the parachute.

Exercises

HOMEWORK HELP

| For Exercises | See Examples |
|---------------|--------------|
| 4–9 | 1 |
| 10, 11 | 2 |

Find the lateral and total surface area of each regular pyramid. Round to the nearest tenth if necessary.

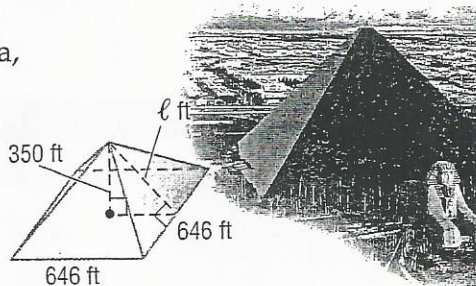


10. **ARCHITECTURE** The Transamerica Pyramid in San Francisco is shaped like a square pyramid. It has a slant height of 856.1 feet and each side of its base is 145 feet. Find the lateral area of the building.

11. **ROOFS** A pyramid-shaped roof has a slant height of 16 feet and its square base is 40 feet wide. How much roofing material is needed to cover the roof?

12. A square pyramid has a lateral area of 107.25 square centimeters and a slant height of 8.25 centimeters. Find the length of each side of its base.

13. **GLASS** The Luxor Hotel in Las Vegas, Nevada, is a pyramid-shaped building standing 350 feet tall and covered with glass. Its base is a square with each side 646 feet long. Find the surface area of the glass on the Luxor. (*Hint: Use the Pythagorean Theorem to find the pyramid's slant height l .*)



EXTRAPRACTICE

See pages 714, 734.

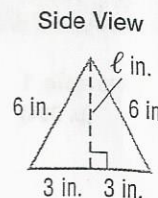
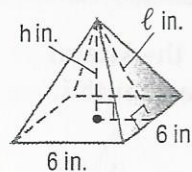
Math online

Self-Check Quiz at tx.msmath3.com

H.O.T. Problems

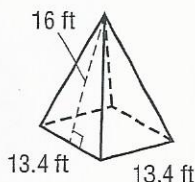
CHALLENGE For Exercises 14–16, use the drawings of the pyramid shown, whose lateral faces are equilateral triangles.

14. Find the measure of the slant height ℓ .
15. Use the slant height to find the height h of the pyramid.
16. Find the volume and surface area of the pyramid.
17. **OPEN ENDED** Draw a square pyramid, giving measures for its slant height and base side length. Then find its lateral area. Justify your answer.
18. **WRITING IN MATH** Explain how the slant height and the height of a pyramid are different.



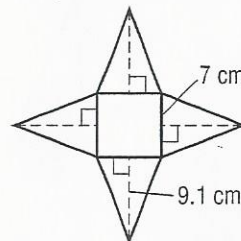
TEST PRACTICE

19. Which is the best estimate for the surface area of the pyramid?



- | | |
|-----------------------|-----------------------|
| A 107 ft ² | C 429 ft ² |
| B 180 ft ² | D 608 ft ² |

20. The net of a paperweight is shown below. Which is closest to the lateral surface area of the paperweight?



- | | |
|----------------------|-----------------------|
| F 32 cm ² | H 127 cm ² |
| G 49 cm ² | J 176 cm ² |

Spiral Review

21. **GEOMETRY** Find the surface area of a cylinder whose diameter is 22 feet and whose height is 7.5 feet. (Lesson 7-7)
22. **TOYS** A beach ball has a diameter of 12 inches. What is the volume of air needed to inflate the ball? (Lesson 7-6)
23. **SCHOOL** Refer to the table at the right. What percent of students are enrolled in public school? (Lesson 5-3)

| U.S. School Enrollment (K–12) | |
|-------------------------------|-------------------|
| Type | Number (millions) |
| Private | 6.3 |
| Public | 48.0 |

Source: National Center for Education Statistics

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each proportion. (Lesson 4-2)

- | | | | |
|------------------------------------|---------------------------------|-----------------------------------|------------------------------------|
| 24. $\frac{16}{n} = \frac{12}{40}$ | 25. $\frac{3}{5} = \frac{x}{8}$ | 26. $\frac{a}{13} = \frac{7}{39}$ | 27. $\frac{10}{26} = \frac{30}{w}$ |
|------------------------------------|---------------------------------|-----------------------------------|------------------------------------|